Networks and Helpers

Yossef Steinberg

IZS 2024

Joint works with Dor Itzhak, Wasim Huleihel

Outline

- Basic cooperation models (networks)
  - The relay channel (RC)
  - The BC with conferencing decoders
  - The multiple access channel (MAC) with conferencing/cribbing encoders
- Unreliable helpers
  - BC
  - Relay + BC (RBC)
  - Relay with unreliable link
  - MAC with unreliable cribbing

(the cost of robustness: rates, dimensionality)

Related models

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- Feedback, generalized feedback (noisy cribbing)
- Mix of the above

Related models - recent works I

A very brief overview of works on cognitive systems, helpers in state dependent channels...

- The Heegard & El Gamal problem [Heegard & El Gamal IT 1983]
- The state dependent BC with conferencing decoders [Dikstein, Permuter, S. IT 2016]
- State-dependent IC with one sided cribbing [Bross & S. ISIT 2011]
- IC with generalized feedback (noisy cribbing) [Bross, S. & Tinguely IT 2013]
- Cognitive IC with secrecy [Liang, Somekh-Baruch, Poor, Shamai, Verdu IT 2009]
- Cognitive cooperative MAC with cribbing and p2p interference [Shimonovich, Somekh-Baruch, Shamai ITW 2013]

Related models - recent works II

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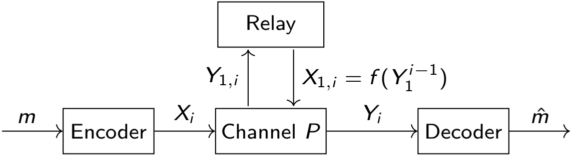
- Cognitive IC with SI [Somekh-Baruch, Shamai, Verdu ISIT 2008]
- Conferencing MAC with states [Permuter, Shamai, Somekh-Baruch, Eilat 2010]
- Cooperative MAC with states at one transmitter [Somekh-Baruch, Shamai, Verdu, IT 2008]
- Message and state cooperation in MAC [Permuter, Shamai, Somekh-Baruch IT 2011]
- MAC with cribbing, feedback, and causal SI [Bracher, Lapidoth, S ITW 2012, Bracher Lapidoth IT 2014]
- SD MAC with states available at a cribbing encoder [Bross Lapidoth Eilat 2010]

Cooperation in networks

The relay channel (van der Meulen 1971)

A double input, double output memoryless channel $P(y, y_1|x, x_1)$:



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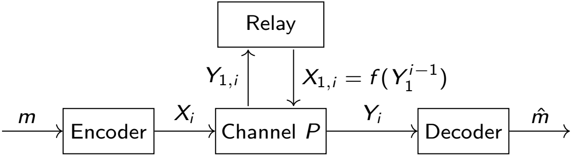

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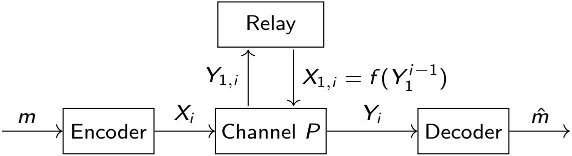

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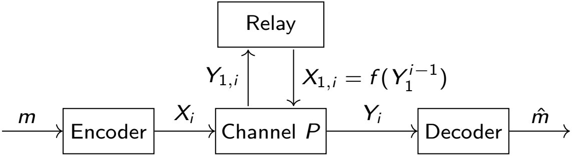

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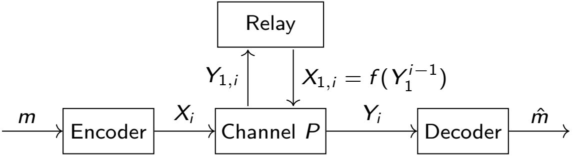

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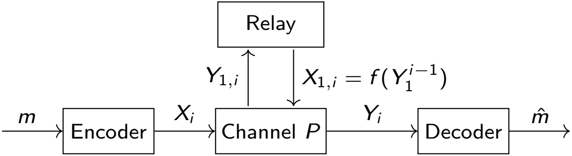

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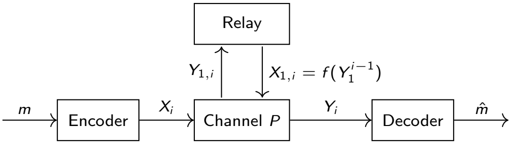

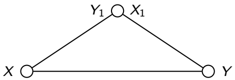

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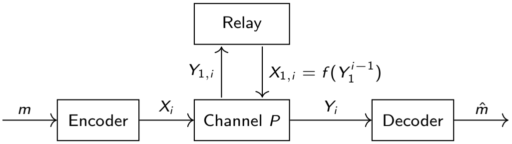

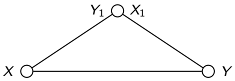

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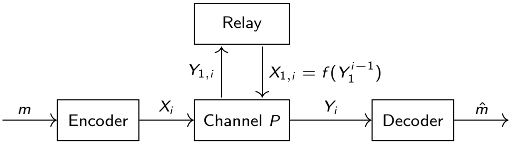

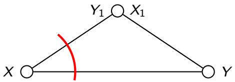

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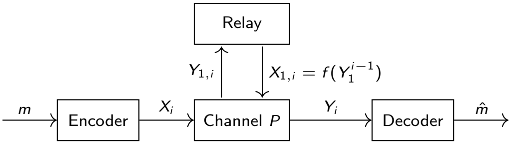

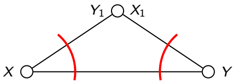

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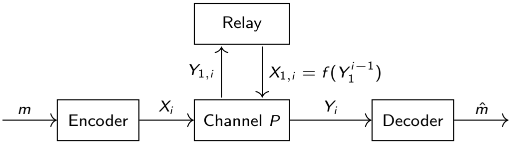

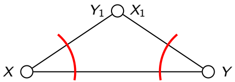

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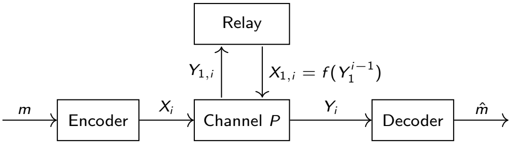

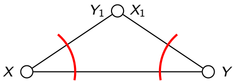

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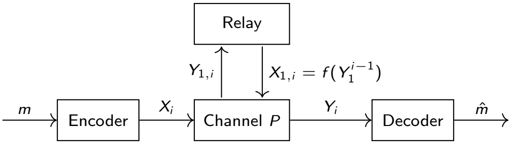

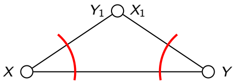

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Coding techniques: Block-Markov (BM) + binning, forward decoding, or: BM + backward decoding (Willems & van der Muelen 85).

The RC P is *degraded* if

$$P(y, y_1|x, x_1) = P(y_1|x, x_1)P(y|x_1, y_1)$$

or:

$$X \multimap (X_1, Y_1) \multimap Y$$

Cooperation in networks

Degraded and reversely degraded RCs

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and *reversely degraded* if (switch the roles of y and y_1...)

$$P(y, y_1|x, x_1) = P(y|x, x_1)P(y_1|x_1, y)$$

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Cover & El Gamal (79) showed that for the degraded RC

$$C = \max_{P_{X,X_1}} \min\{I(X, X_1; Y), I(X; Y_1|X_1)\} \tag{1}$$

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Cooperation in networks

Capacity of degraded and reversely degraded RCs

Cover & El Gamal (79) showed that for the degraded RC

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Only physical degradedness is defined. Stochastic degradation does not yield results beyond the general RC, due to the inherent cooperation between the relay and destination.

The relay channel

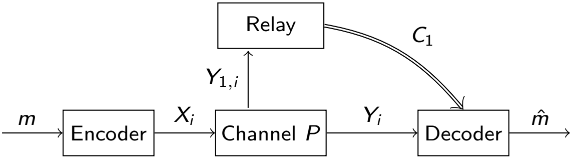
Outer bound for general RC:

$$C \le \max_{P_{X,X_1}} \min\{I(X, X_1; Y), I(X; YY_1|X_1)\} \tag{3}$$

Proof - cutset bound, or with decomposition techniques.

- Degraded RC:
  - Achievability is based on decode and forward at the relay, block-Markov (BM) coding, binning and forward decoding, or BM coding and backward decoding (Willems & v. d. Meulen 85).
  - Converse - use degradedness in (3).
- Reversely degraded RC:
  - Achievability via direct transmission, X_1 *opens* the channel to Y
  - Converse - use rev. deg. in (3).

The primitive relay channel (PRC)



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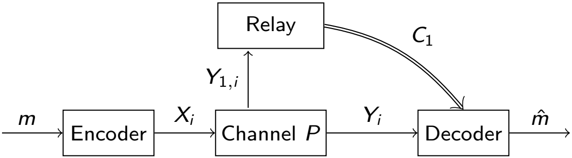

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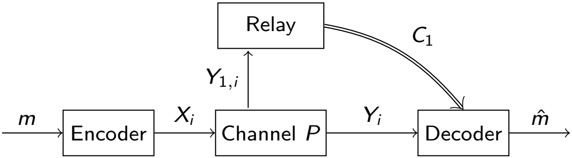

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$$C = \max_{P_X} \min \{I(X;Y) + C_1, I(X;Y_1)\}$$

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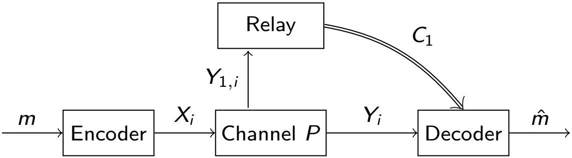

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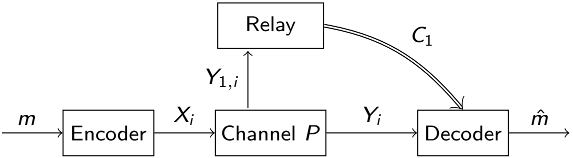

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[Kim & Cover ISIT 2007, Kim Allerton 2007, IT 2008]

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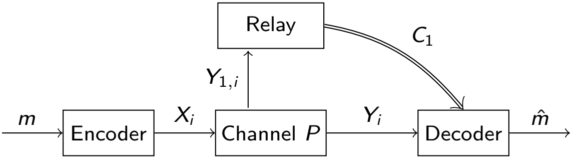

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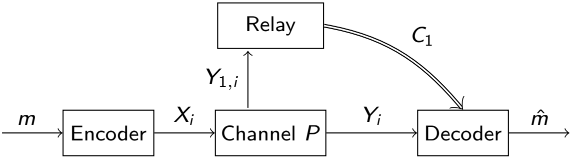

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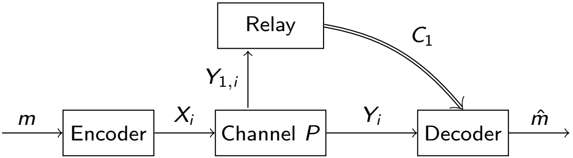

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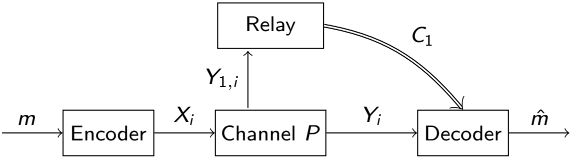

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Causality constraints are meaningless

General RC models

The RC is solved for

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- Reversely degraded
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- Semideterministic RC ($Y_1 = g(X, X_1)$)
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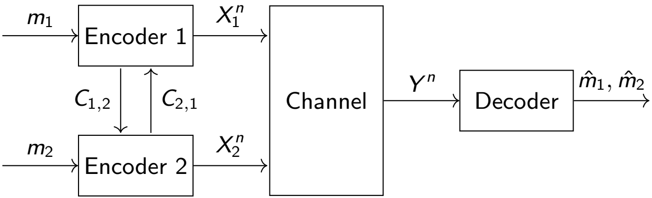
- Orthogonal receiver components (RFD for Gaussian models) This is the PRC without degradedness or deterministic relay assumptions

Lower bounding techniques:

- Decode and forward
- Partial decode and forward
- Compress and forward

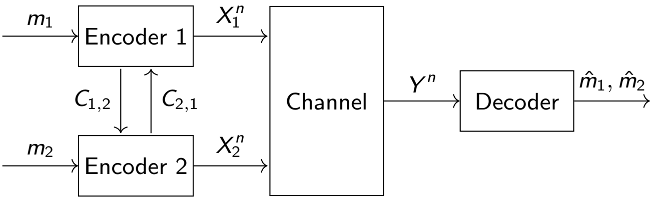
MAC with conferencing encoders

(Willems 83)



- The encoders can exchange information before transmission, via links of limited capacity.
- Multiple rounds allowed

MAC with conferencing encoders



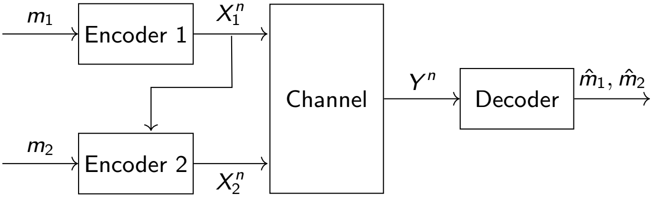
The capacity region:

$$
\begin{aligned}
\mathcal{C} = \{(R_1, R_2): \quad R_1 &\leq I(X_1; Y|X_2, U) + C_{1,2} \\
R_2 &\leq I(X_2; Y|X_1, U) + C_{2,1} \\
R_1 + R_2 &\leq \min\{I(X_1 X_2; Y|U) + C_{1,2} + C_{2,1}, I(X_1 X_2; Y) \\
\text{for some} \quad & P_U P_{X_1|U} P_{X_2|U} P_{Y|X_1 X_2}\}
\end{aligned}
$$

One conferencing round suffices.

MAC with cribbing encoders

Willems & van der Meulen IT 85



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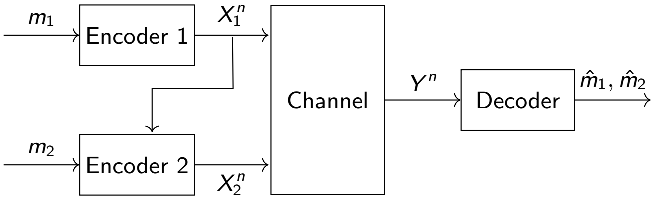

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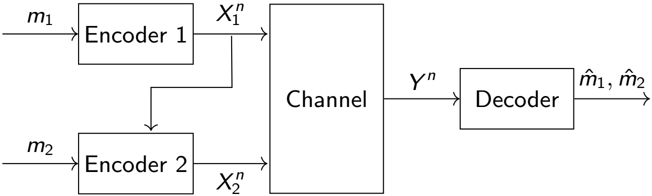

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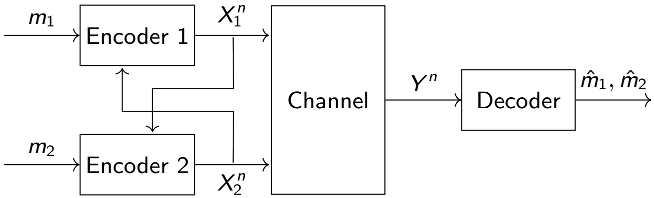

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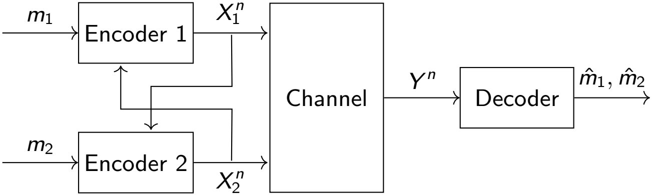
- Various forms:
  - strictly causal, causal, or non causal cribbing
  - one sided, or (consistent) two-sided.

MAC with cribbing encoders

Willems & van der Meulen IT 85



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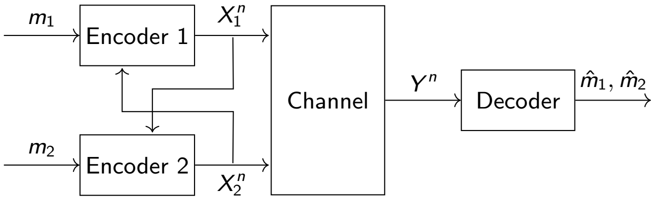

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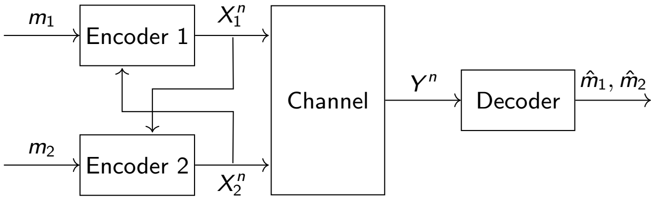

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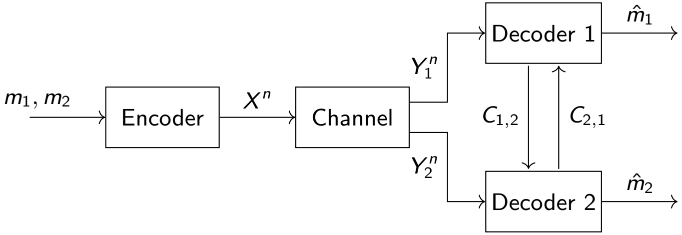
BM coding + backward decoding

The BC with cooperating decoders

Daborah & Servetto 04, 06. Liang & Veeravalli 04, 07.



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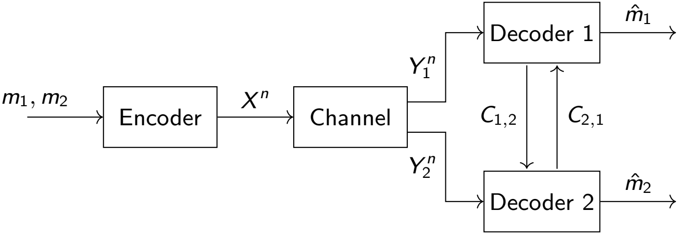

Assume a physically degraded BC:

$$P_{Y_1,Y_2|X} = P_{Y_1|X} P_{Y_2|Y_1}$$

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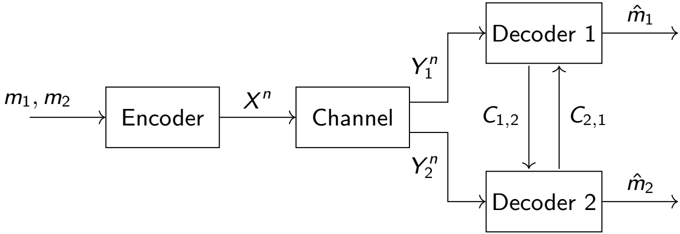

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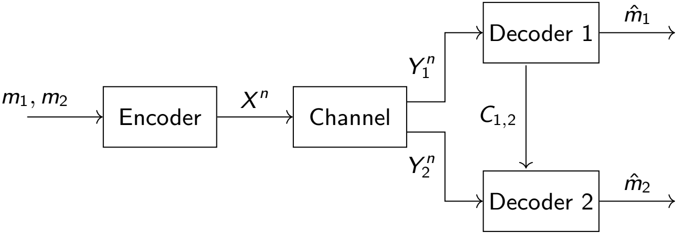
$C_{2,1}$ does not increase capacity

The degraded BC (DBC) with cooperating decoders



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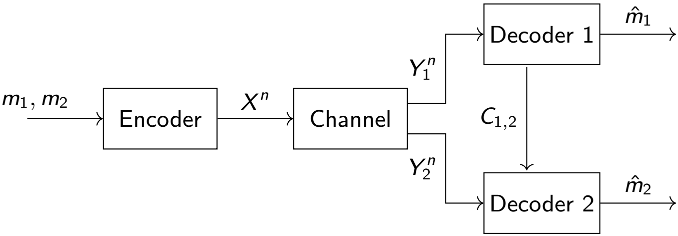

The capacity region (Dabora & Servetto 04):

$$R_2 \leq \min\{I(U;Y_2) + C_{1,2}, I(U;Y_1)\}$$
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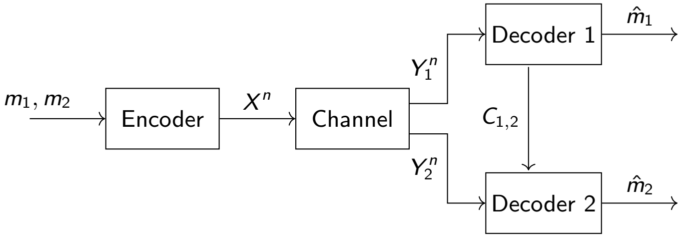

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Coding technique - classical BC + block-Markov coding for the conference messages.
(also: Liang & Veeravalli 07, Liang 05)

The DBC with cooperating decoders

Alternative technique - binning

Suggested originally for the same setting with states (Dikstein, Permuter & S, Allerton 13, IT 16). When specialized to the case of no states, the DPS region reduces to

$$\begin{aligned} R_2 &\leq I(U;Y_2) + C_{1,2} \\ R_1 &\leq I(X;Y_1|U) \\ R_1 + R_2 &\leq I(X;Y_1) \end{aligned}$$

Equivalent to D&S region

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The DBC with cooperating decoders

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  - In its current form, (6) resembles the capacity region of the general BC with degraded message sets.

The DBC with cooperating decoders

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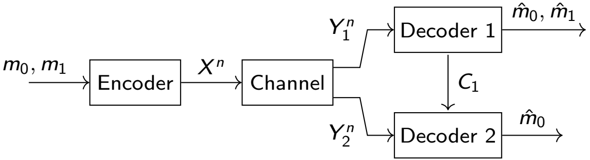
- General BC with degraded message sets - one receiver is *required* to decode both messages. Is d&f optimal in the presence of conferencing?

Degraded message sets

The general BC with degraded message sets and conferencing

(S. ISIT 15)



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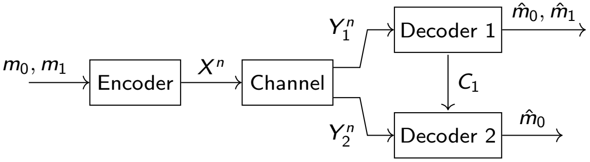

The capacity region is the set of all rate pairs (R_0, R_1) satisfying

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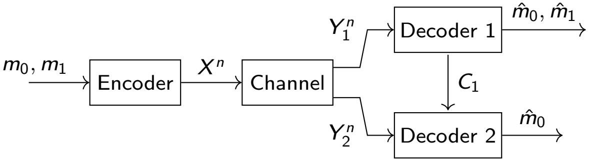

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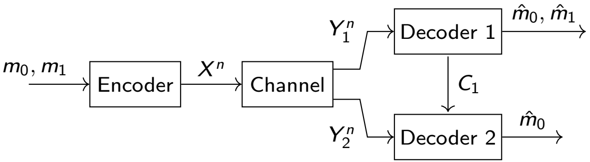

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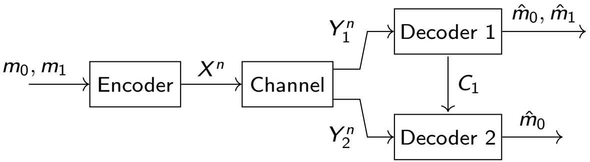

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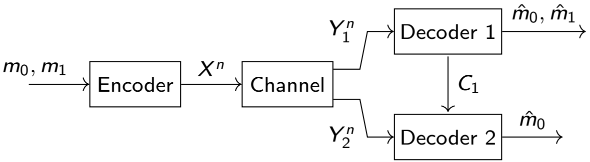

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- Capacity region characterization resembles that of the DBC
- Superposition coding + d&f is optimal. Converse is more involved than that for the DBC.

Unreliable cooperation links

- Cooperation improves performance, but
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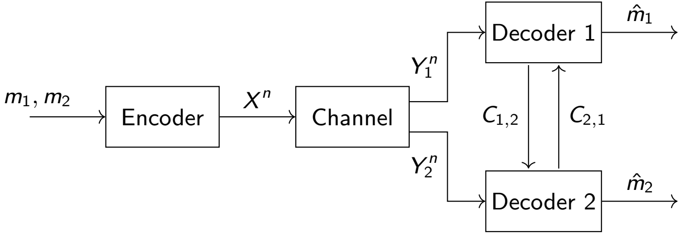
- Modern ad-hoc networks are dynamic, so
  - availability of cooperation links cannot be guaranteed a priori
  - often users cannot be informed about their availability.

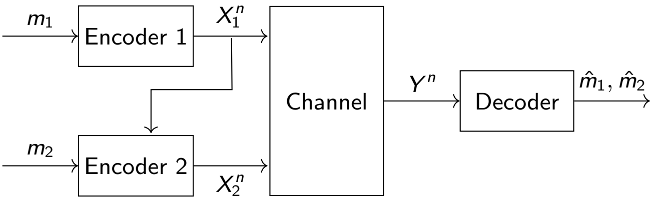
Unreliable cooperation links

- BC with conferencing decoders: if the conference links are not present, the encoder is not aware of it



Unreliable cooperation links

- MAC with cribbing encoder (one sided): Encoder 1 does not know whether Encoder 2 cribs or not.



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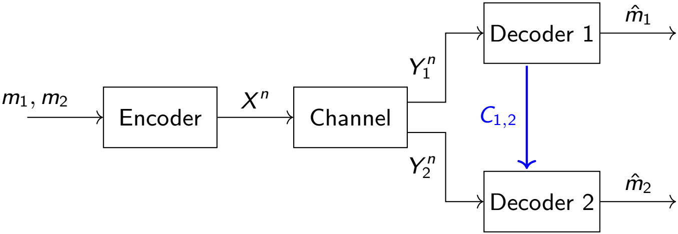
$\Longrightarrow$ We need robust coding schemes that exploit the cooperation links if they are present, but can still operate - possibly at reduced decoding rates - if they are absent.

The DBC with unreliable helper

[S. ISIT 14, Huleihel & S. IT 17]



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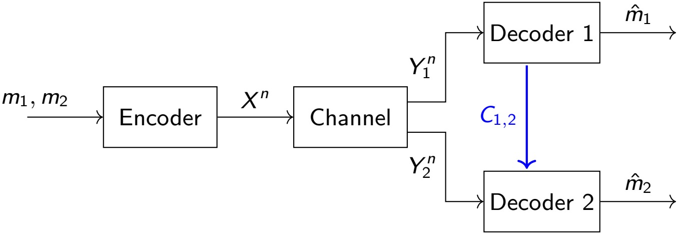

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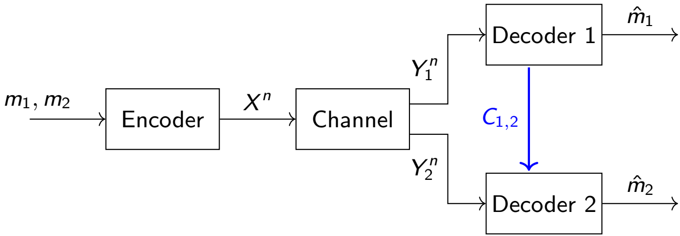

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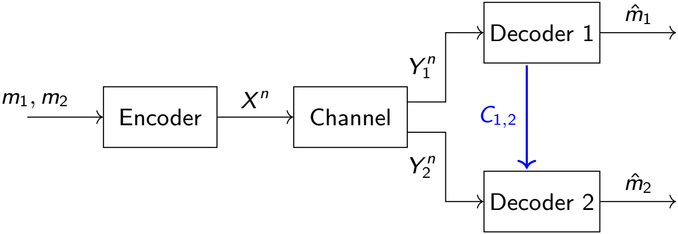

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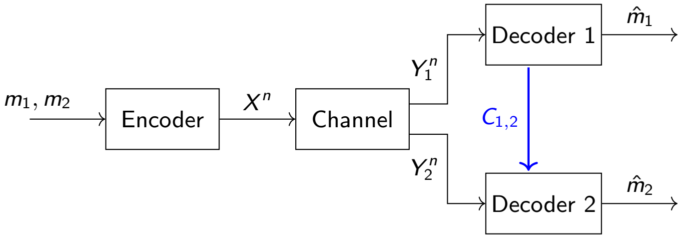

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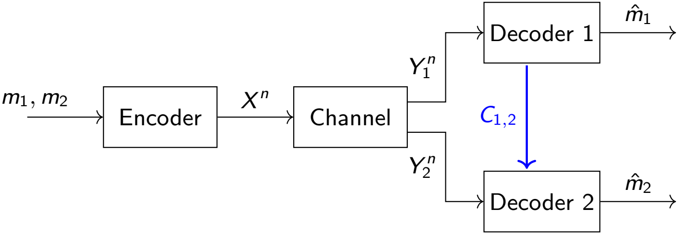

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The capacity region $\mathcal{C}(C_{1,2})$ is the set of all achievable triples (R_1, R_2, R_2') with unreliable cooperation link of capacity $C_{1,2}$.

The DBC with unreliable helper

Capacity region

[S. ISIT 14, Huleihel & S. IT 17]

Theorem

The capacity region $\mathcal{C}(C_{1,2})$ of the DBC $P_{Y_1|X}P_{Y_2|Y_1}$ is the set of all (R_1, R_2, R_2') satisfying

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\begin{aligned}
R_2 &\leq I(U; Y_2) \\
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R_1 &\leq I(X; Y_1|U, V)
\end{aligned}
$$

for some joint distribution of the form

$$
P_{U,V,X,Y_1,Y_2} = P_{U,V}P_{X|U,V}P_{Y_1|X}P_{Y_2|Y_1}.
$$

The DBC with unreliable helper

Comparison between regions

When $C_{1,2}$ is always present

$$R_2 \leq \min\{I(U;Y_2) + C_{1,2}, I(U;Y_1)\}$$
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The DBC with unreliable helper

Coding technique

Two layers of superposition + binning:

The DBC with unreliable helper

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- X - coding for User 1. Generated by $P_{X|U,V}$.

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- X - coding for User 1. Generated by $P_{X|U,V}$.
- User 1 decodes X, U, V. User 2 decodes U.
- If $C_{1,2}$ is present, User 1 sends the bin number of V to User 2.
  User 2 decodes V.
  This succeeds with high prob. if

$$R_2' - C_{1,2} < I(V; Y_2|U)$$

The DBC with unreliable helper

Comparison between regions

$C_{1,2}$ reliable

$$R_2 \leq \min\{I(U;Y_2) + C_{1,2}, I(U;Y_1)\}$$

$$R_1 \leq I(X;Y_1|U)$$

$$U - X - Y_1 - Y_2.$$

$C_{1,2}$ unreliable

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$$(UV) - X - Y_1 - Y_2.$$

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Comparison between regions

$C_{1,2}$ reliable

$$R_2 \leq \min\{I(U;Y_2)+C_{1,2}, I(U;Y_1)\}$$

$$R_1 \leq I(X;Y_1|U)$$

$$U - X - Y_1 - Y_2.$$

$C_{1,2}$ unreliable

$$R_2 \leq I(U;Y_2)$$

$$R_2' \leq \min\{I(V;Y_2|U)+C_{1,2}, I(V,Y_1|U)\}$$

$$R_1 \leq I(X;Y_1|U,V)$$

$$(UV) - X - Y_1 - Y_2.$$

U null - coincides with reliable $C_{1,2}$

The DBC with unreliable helper

Comparison between regions

$C_{1,2}$ reliable

$$R_2 \leq \min\{I(U;Y_2) + C_{1,2}, I(U;Y_1)\}$$

$$R_1 \leq I(X;Y_1|U)$$

$$U - X - Y_1 - Y_2.$$

$C_{1,2}$ unreliable

$$R_2 \leq I(U;Y_2)$$

$$R_2' \leq \min\{I(V;Y_2|U) + C_{1,2}, I(V,Y_1|U)\}$$

$$R_1 \leq I(X;Y_1|U,V)$$

$$(UV) - X - Y_1 - Y_2.$$

U null - coincides with reliable $C_{1,2}$

V null - coincides with $C_{1,2} = 0$.

The DBC with unreliable helper

Comparison between regions

$C_{1,2}$ reliable

$$R_2 \leq \min\{I(U;Y_2) + C_{1,2}, I(U;Y_1)\}$$

$$R_1 \leq I(X;Y_1|U)$$

$$U - X - Y_1 - Y_2.$$

$C_{1,2}$ unreliable

$$R_2 \leq I(U;Y_2)$$

$$R_2' \leq \min\{I(V;Y_2|U) + C_{1,2}, I(V,Y_1|U)\}$$

$$R_1 \leq I(X;Y_1|U,V)$$

$$(UV) - X - Y_1 - Y_2.$$

U null - coincides with reliable $C_{1,2}$
V null - coincides with $C_{1,2} = 0$.

- The pessimistic and optimistic approaches are special cases.

The DBC with unreliable helper

Comparison between regions

$C_{1,2}$ reliable

$$R_2 \leq \min\{I(U;Y_2) + C_{1,2}, I(U;Y_1)\}$$

$$R_1 \leq I(X;Y_1|U)$$

$$U - X - Y_1 - Y_2.$$

$C_{1,2}$ unreliable

$$R_2 \leq I(U;Y_2)$$

$$R_2' \leq \min\{I(V;Y_2|U) + C_{1,2}, I(V,Y_1|U)\}$$

$$R_1 \leq I(X;Y_1|U,V)$$

$$(UV) - X - Y_1 - Y_2.$$

U null - coincides with reliable $C_{1,2}$

V null - coincides with $C_{1,2} = 0$.

- The pessimistic and optimistic approaches are special cases.
- Cost of Robustness is reflected in the region

The DBC with unreliable helper

Comparison between regions

$C_{1,2}$ reliable

$$R_2 \leq \min\{I(U;Y_2) + C_{1,2}, I(U;Y_1)\}$$

$$R_1 \leq I(X;Y_1|U)$$

$$U - X - Y_1 - Y_2.$$

$C_{1,2}$ unreliable

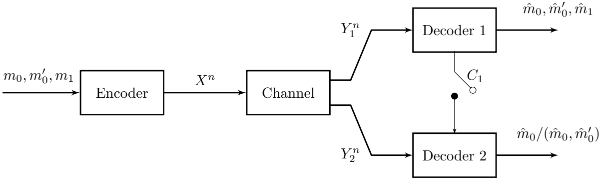
$$R_2 \leq I(U;Y_2)$$
$$R_2' \leq \min\{I(V;Y_2|U) + C_{1,2}, I(V,Y_1|U)\}$$
$$R_1 \leq I(X;Y_1|U,V)$$

$$(UV) - X - Y_1 - Y_2.$$

U null - coincides with reliable $C_{1,2}$
V null - coincides with $C_{1,2} = 0$.

- The pessimistic and optimistic approaches are special cases.
- Cost of Robustness is reflected in the region
- Added dimension - 2 users, 3 dim. region.

[Itzhak & S. IT 21]



- m_1 - a private message to User 1
- Only User 2 benefits from the conference link:
  - R_0 - rate decoded whether the conference link is present or not.
  - $R_0^{'}$ - extra rate decoded if the conference link is present.
- User 1 always decodes all three messages.

The capacity region $\mathcal{C}$ - the set of all achievable triples $(R_0, R_0^{'}, R_1)$.

General BC

The BC with Degraded Message Sets and Unreliable Conference

[Itzhak & S. IT 21]

Theorem

The capacity region is the set of all triples (R_0, R_0', R_1) *satisfying:*

$$\begin{aligned} R_0 &\leq I\left(U; Y_2\right) \\ R_0' &\leq I\left(V; Y_2 | U\right) + C_1 \\ R_1 &\leq I\left(X; Y_1 | UV\right) \\ R_0' + R_1 &\leq I\left(X; Y_1 | U\right) \\ R_0 + R_0' + R_1 &\leq I\left(X; Y_1\right) \end{aligned}$$

for some joint distribution $p\left(u, v, x\right) p\left(y_1, y_2 | x\right)$.

Denote this region by $\mathcal{R}^i$.

Genaral BC

The BC with degraded message sets and unreliable conference

Remarks.

- The capacity region depends only on the channel conditional marginals $P(y_1|x)$ and $P(y_2|x)$.
- The message sets degradedness is crucial. Decoder 1 decodes m_0' and then helps decoder 2 based on its decision (d&f is optimal)
- Coding technique is similar to that for the DBC with unreliable helper.
- Converse is much more involved.

General BC

Method of proof - outer bound

Let $\mathcal{R}^o$ be the set of all triples (R_0, R_0', R_1) satisfying:

$$
\begin{aligned}
R_0 &\leq I(U; Y_2) \\
R_0 + R_0' &\leq I(UV; Y_2) + C_1 \\
R_0 + R_0' + R_1 &\leq I(U; Y_2) + I(X; Y_1|U) \\
R_0 + R_0' + R_1 &\leq I(UV; Y_2) + C_1 + I(X; Y_1|UV) \\
R_0 + R_0' + R_1 &\leq I(X; Y_1)
\end{aligned}
$$

for some joint distribution $p(u, v, x)\, p(y_1, y_2|x)$.

$\mathcal{R}^o$ is an outer bound for the capacity region.

General BC

Method of proof - equivalence of $\mathcal{R}^o$ and $\mathcal{R}^i$

The achievable region introduced in the theorem $\mathcal{R}^i$:

$$\begin{aligned} R_0 &\leq I(U; Y_2) \\ R_0' &\leq I(V; Y_2|U) + C_1 \\ R_1 &\leq I(X; Y_1|UV) \\ R_0' + R_1 &\leq I(X; Y_1|U) \\ R_0 + R_0' + R_1 &\leq I(X; Y_1) \end{aligned}$$

The outer bound $\mathcal{R}^o$:

$$\begin{aligned} R_0 &\leq I(U; Y_2) \\ R_0 + R_0' &\leq I(UV; Y_2) + C_1 \\ R_0 + R_0' + R_1 &\leq I(U; Y_2) + I(X; Y_1|U) \\ R_0 + R_0' + R_1 &\leq I(UV; Y_2) + C_1 + I(X; Y_1|UV) \\ R_0 + R_0' + R_1 &\leq I(X; Y_1) \end{aligned}$$

General BC

Method of proof - equivalence of $\mathcal{R}^o$ and $\mathcal{R}^i$

Equivalence proof - using methods from polytope theory and convex analysis:

- $\mathcal{R}^i \subseteq \mathcal{R}^o$ as each polytope in $\mathcal{R}^i$ is also in $\mathcal{R}^o$.
- $\mathcal{R}^o \subseteq \mathcal{R}^i$ - proof as follows
  - any polytope induced from $p(u, v, x)$ in $\mathcal{R}^o$ is defined by its own vertices (extreme points).
  - prove that all polytopes vertices in $\mathcal{R}^o$ are also in $\mathcal{R}^i$.
  - use the fact that a compact convex set is the convex hull of its own extreme points.

Gaussian channel

Example - AWGN BC with degraded message sets and unreliable link

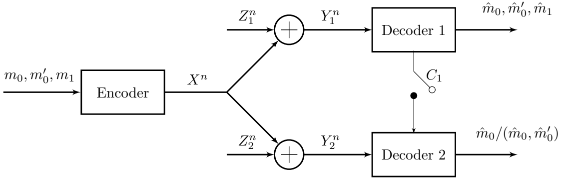
Example

The AWGN BC with degraded message sets and unreliable link:

$$Y_1 = X + Z_1 \qquad Z_1 \sim \mathcal{N}(0, N_1)$$
$$Y_2 = X + Z_2 \qquad Z_2 \sim \mathcal{N}(0, N_2)$$

where $N_2 > N_1$, the noise signals Z_1 and Z_2 are independent and an input power constraint $E\left[X^2\right] \le P$.

- The main channel is stochastically degraded.



Gaussian channel

Example - AWGN BC with degraded message sets and unreliable link

The capacity region for this model is given by the set of all rate triples (R_0, R_0', R_1) satisfying:

$$
\begin{aligned}
R_0 &\leq \mathcal{C}\left(\frac{\alpha_0 P}{N_2 + (\alpha_0' + \alpha_1) P}\right) \\
R_0' &\leq \mathcal{C}\left(\frac{\alpha_0' P}{N_2 + \alpha_1 P}\right) + C_1 \\
R_1 &\leq \mathcal{C}\left(\frac{\alpha_1 P}{N_1}\right) \\
R_0' + R_1 &\leq \mathcal{C}\left(\frac{(\alpha_0' + \alpha_1) P}{N_1}\right)
\end{aligned}
$$

where $\mathcal{C}(x) \triangleq \frac{1}{2}\log(1+x)$ is the capacity of the classical AWGN, $\alpha_0, \alpha_0', \alpha_1 \geq 0$ and $\alpha_0 + \alpha_0' + \alpha_1 = 1$.

Gaussian channel

Example - AWGN BC with degraded message sets and unreliable link

Method of proof

- Apply the capacity theorem
- Converse part via EPI
- For the direct part, use Gaussian channel input.

Cost of robustness

Robustness incurs penalty:

- If the unreliable link is absent,
  we should be able to communicate at a slightly lower rate compared to the case of no link at all.
- If the unreliable link is present,
  we should be able to communicate at a slightly lower rate compared to the case of reliable link present.

Cost of robustness

Comparison with the classical AWGN BC

Classical AWGN BC with degraded message sets (no link)

- R_0 - rate for User 2 (and User 1)
- R_1 - rate solely for User 1

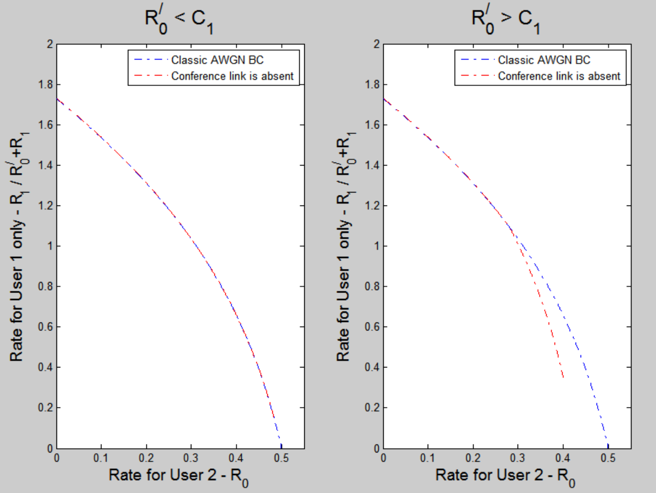
Our AWGN BC model where the link is absent

- 2D capaity region for a constant R_0'
- R_0 - rate for User 2 (and User 1)
- $R_0' + R_1$ - rate decoded only by User 1

Cost of robustness

Comparison with the classical AWGN BC - $R_0^{'}$ constant, link absent

If the link is absent, $R_0^{'} + R_1$ is decoded only by User1:



Cost of robustness

Comparison with the classical AWGN BC - R_0' is constant

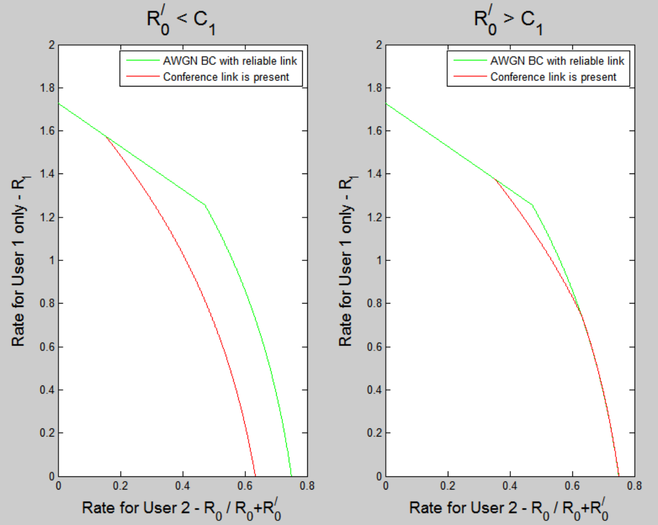
- $R_0' < C_1$:
  there is no rate reduction comparing to the classical model, as we should only use the bit-pipe conference link to communicate the extra message.

- $R_0' > C_1$:
  we actually pay for the robust coding scheme as we consume power $\alpha_0' > 0$ to communicate the extra message.

Cost of robustness

Comparison with the AWGN BC with reliable link - R_0' constant, link present

If the link is present, $R_0 + R_0'$ is dedicated to User2:



Cost of robustness

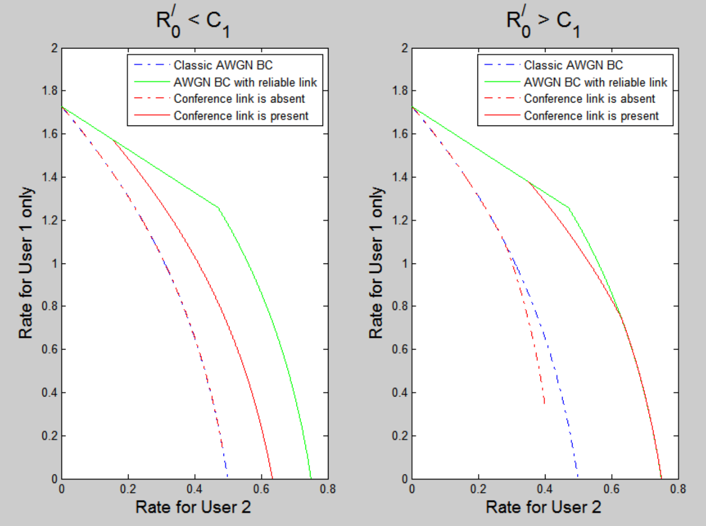
Comparison with the AWGN BC with reliable conference - $R_0^{'}$ is constant

- $R_0^{'} < C_1$:
  here there is actually no intention to fully exploit the conference link. In the case of reliable link all parties do their best to exploit the full capacity of the conference link.

- $R_0^{'} > C_1$:
  still paying for the robust coding scheme, but a reasonable cost.

Cost of robustness

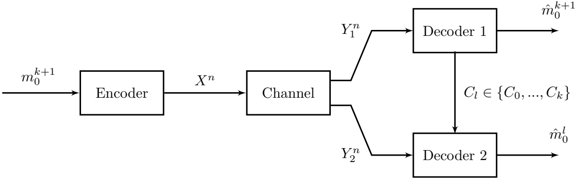
Comparison of all setups for the AWGN BC



Many degrees of uncertainty

The BC with degraded message sets and unreliable conference - k stages

Consider the following generalization



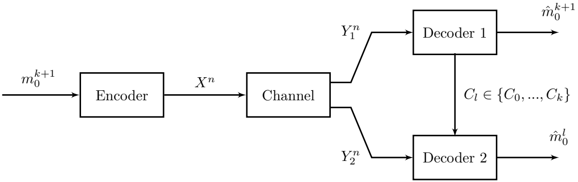
- unreliable conference link with many possible capacity values

$$\{C_0, C_1, ..., C_k\} \qquad 0 = C_0 \leq C_1 \leq ... \leq C_k$$

- the encoder is not aware of the actual conference link capacity, both decoders are aware of it

Many degrees of uncertainty

The BC with degraded message sets and unreliable conference - k stages



There are $k+2$ messages (or rates):

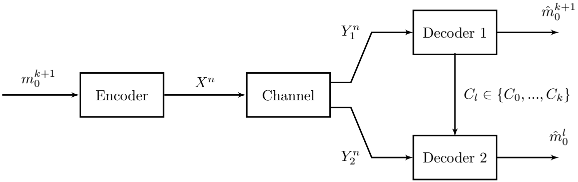
- R_0 - a common rate always decoded by User 2
- $R_1, ..., R_l$ - residual common rates decoded by User 2 if the actual capacity is C_l

- R_{k+1} - a private message to User 1 only

User 1 decodes all the messages independently of the actual conference link capacity.
The capacity region $\mathcal{C}_k$ - the set of all achievable rate tuples $\mathbf{R} = (R_0, ..., R_{k+1})$.

Many degrees of uncertainty

The BC with degraded message sets and unreliable conference - k stages



There are $k+2$ messages (or rates):

- R_0 - a common rate always decoded by User 2
- $R_1, ..., R_l$ - residual common rates decoded by User 2 if the actual capacity is C_l

  \- R_l are residual rates. C_l are cumulative capacities.
- R_{k+1} - a private message to User 1 only

User 1 decodes all the messages independently of the actual conference link capacity.

The capacity region $\mathcal{C}_k$ - the set of all achievable rate tuples $\mathbf{R} = (R_0, ..., R_{k+1})$.

Many degrees of uncertainty

The BC with degraded message sets and unreliable conference - k stages

Let $\mathcal{R}_k$ be the set of all rate vectors $\mathbf{R} = (R_0, R_1, ..., R_{k+1})$ satisfying:

$$\sum_{i=0}^{l} R_i \leq I\left(U_0^l; Y_2\right) + C_l$$

$$l = 0, .., k$$

$$\sum_{i=0}^{k+1} R_i \leq I\left(U_0^m; Y_2\right) + C_m + I\left(X; Y_1 | U_0^m\right)$$

$$m = -1, 0, .., k$$

for some joint distribution $p\left(u_0, .., u_k, x\right) p\left(y_1, y_2 | x\right)$. Note that $U_0^{-1} = \emptyset$ and $C_{-1} = 0$ by convention.

Many degrees of uncertainty

The BC with degraded message sets and unreliable conference - k Stages

Theorem

For the k-stages model, the capacity region is given by

$$\mathcal{C}_k = \mathcal{R}_k$$

- The proof uses polytope theory in the direct and converse parts, and Csiszars' sum identity.

Example

The capacity region for this model is given by the set of all rate tuples $\mathbf{R} = (R_0, R_1, ..., R_{k+1})$ satisfying:

$$\sum_{i=0}^{l} R_i \leq \mathcal{C}\left(\frac{\beta_l P}{N_2 + \bar{\beta}_l P}\right) + C_l$$

$$\sum_{i=0}^{k+1} R_i \leq \mathcal{C}\left(\frac{\beta_l P}{N_2 + \bar{\beta}_l P}\right) + C_l + \mathcal{C}\left(\frac{\bar{\beta}_l P}{N_1}\right)$$

$$l = 0, .., k$$

where $0 \leq \beta_0 \leq ... \leq \beta_k \leq 1$ and $\bar{\beta} \triangleq 1 - \beta$.
Alternative representation with α-parameters where $\beta_l = \sum_{i=0}^{l} \alpha_i$.

Thoughts on the performance criterion

- In the models above, users do not know a priori if cooperation will take place.
- They have to decide in what rates they want to operate under each of the scenarios

$$\Longrightarrow \text{region in } \mathbb{R}^{k+2}$$

Only two users, but the dimension increases with k.

- Regions of different channels are hard to compare (complicated cap. region, too many parameters to decide on a priori)

Thoughts on the performance criterion

We seek a (scalar) performance criterion, that

- measures the cooperativeness of the channel
- dictates in what rates to operate

Average rate criterion

Criterion definition

Consider the situation where the conference link capacity is random.

- C_l occurs w.p. q_l. Constant during transmission.
- Still require that the system operates in any realization of C_l.

The average common rate

$$R_E \triangleq \sum_{l=0}^{k} q_l R_{T,l},$$

where

$$R_{T,l} \triangleq \sum_{i=0}^{l} R_i.$$

Average rate criterion

Criterion definition

Our goal: maximize R_E, subject to a constraint on the private rate

$$C_E(r) \triangleq \max_{\mathbf{R}\in\mathcal{C}_k\,,\, R_{k+1}\geq r} \sum_{l=0}^{k} q_l R_{T,l}$$

- System has to operate under ANY realization of C_l, thus $\mathbf{R} \in \mathcal{C}_k$.

  $\Longrightarrow$ *A constrained maximization problem*

- Measures the cooperativeness of the channel - the maximal average rate User 2 decodes, given the system resources allocated for the cooperation.

Average rate criterion

The Gaussian channel

Example

The AWGN BC with $k = 1$

- The conference link is present with capacity C_1 with prob. q
- The conference link is absent with probability $\bar{q} = 1 - q$
- The average common rate

$$R_E = \bar{q} R_0 + q(R_0 + R_0^{'}).$$

Solving the problem with a constraint r on R_1 becomes simpler after solving it with $r = 0$.

Average rate criterion

The Gaussian channel

The achievable region when choosing $R_1 = 0$

$$R_0 \leq \mathcal{C}\left(\frac{\alpha P}{N_2 + \bar{\alpha} P}\right)$$

$$R_0' \leq \min\left\{\mathcal{C}\left(\frac{\bar{\alpha} P}{N_2}\right) + C_1, \mathcal{C}\left(\frac{\bar{\alpha} P}{N_1}\right)\right\}$$

where

$$\mathcal{C}(x) = \frac{1}{2}\log(1 + x),$$

$\alpha \in [0, 1]$ and $\bar{\alpha} = 1 - \alpha$.

Average rate criterion

The Gaussian channel

Two interesting cases:

- $C_1 < C'$
- $C_1 \geq C'$

where

$$C' \triangleq \mathcal{C}\left(\frac{P}{N_1}\right) - \mathcal{C}\left(\frac{P}{N_2}\right).$$

Average rate criterion

The Gaussian channel : $C_1 < C'$

$$C_E = \begin{cases} \mathcal{C}\left(\frac{P}{N_2}\right) & 0 \le q \le \lambda \\ \mathcal{C}\left(\frac{P}{N_2}\right) + \frac{1}{2}\mathcal{D}\left(q \,\|\, \lambda\right) & \lambda < q \le \mu \\ \mathcal{C}\left(\frac{P}{N_2}\right) + qC_1 + \frac{1}{2}\bar{q}\log\left(\frac{\bar{\mu}}{\bar{\lambda}}\right) & \mu < q \le 1 \end{cases}$$

where $\lambda \triangleq \frac{N_1}{N_2}$, $\mu \triangleq \frac{N_1}{N_2}2^{2C_1}$ and $\mathcal{D}\left(q \,\|\, \lambda\right)$ is the KL divergence.
Note that $0 < \lambda < \mu < 1$.

$\Longrightarrow$ When $q \le \frac{N_1}{N_2}$, it is better not to use the link at all.

Average rate criterion

The Gaussian channel : $C_1 < C'$

$$C_E = \begin{cases} \mathcal{C}\left(\frac{P}{N_2}\right) & 0 \leq q \leq \lambda \\ \mathcal{C}\left(\frac{P}{N_2}\right) + \frac{1}{2}\mathcal{D}\left(q \parallel \lambda\right) & \lambda < q \leq \mu \\ \mathcal{C}\left(\frac{P}{N_2}\right) + qC_1 + \frac{1}{2}\bar{q}\log\left(\frac{\bar{\mu}}{\bar{\lambda}}\right) & \mu < q \leq 1 \end{cases}$$

where $\lambda \triangleq \frac{N_1}{N_2}$, $\mu \triangleq \frac{N_1}{N_2}2^{2C_1}$ and $\mathcal{D}\left(q \parallel \lambda\right)$ is the KL divergence. Note that $0 < \lambda < \mu < 1$.

$\Longrightarrow$ When $q \leq \frac{N_1}{N_2}$, it is better not to use the link at all.

(when q is too low, the presence of the link does not justify the penalty incurred by the robust coding scheme)

Average rate criterion

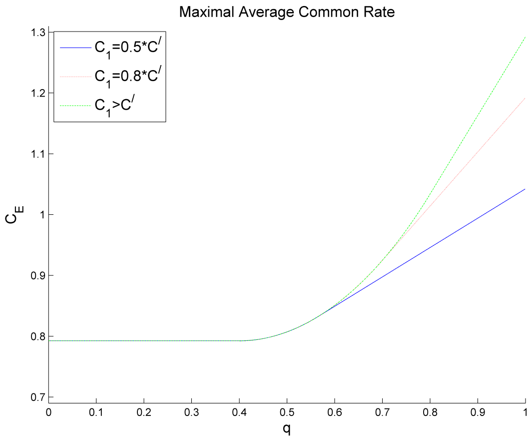
The Gaussian channel : $C_1 \geq C'$

$$C_E = \begin{cases} \mathcal{C}\left(\frac{P}{N_2}\right) & 0 \leq q \leq \lambda \\ \mathcal{C}\left(\frac{P}{N_2}\right) + \frac{1}{2}\mathcal{D}\left(q \parallel \lambda\right) & \lambda < q \leq \frac{N_1+P}{N_2+P} \\ q\mathcal{C}\left(\frac{P}{N_1}\right) & \frac{N_1+P}{N_2+P} < q \leq 1 \end{cases}$$

- No dependency on the conference link capacity C_1
- Note that when $q \leq \frac{N_1}{N_2}$, the presence of the link for ANY C_1 does not justify the penalty incurred by the robust coding scheme

Average rate criterion

The Gaussian channel



$$R_1 = 0, \quad P = 5, \quad N_1 = 1, \quad N_2 = 2.5, \quad C' = 0.5$$

Average rate criterion

The Gaussian channel

Conclusions:

- If probability of the cooperation to be present is small,
$$q \leq \frac{N_1}{N_2},$$
it is best to avoid cooperation a priori.
- There is no reason to provide a conference link capacity greater than
$$\mathcal{C}\left(\frac{P}{N_1}\right) - \mathcal{C}\left(\frac{P}{N_2}\right).$$

Average rate criterion

The Gaussian channel

Maximize the average common rate, subject to $R_1 \geq r > 0$

- choosing $R_1 = r$ maximize R_E
- define P_1 such that $R_1 = r = \frac{1}{2}\log\left(1 + \frac{P_1}{N_1}\right)$
- the maximal average common rate $C_E(r)$, is the same function C_E for the problem with no constraint on R_1, with the modified parameters:

$$
\begin{aligned}
P' &= P - P_1 \\
N_1' &= N_1 + P_1 \\
N_2' &= N_2 + P_1
\end{aligned}
$$

Average rate criterion

The Gaussian channel

Solved for:

- Average rate with $k > 1$

Average rate criterion

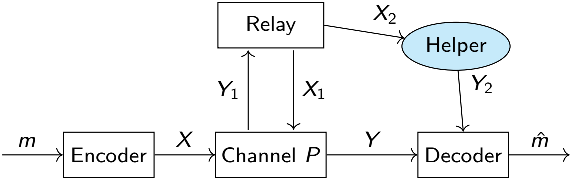
The Gaussian channel

Solved for:

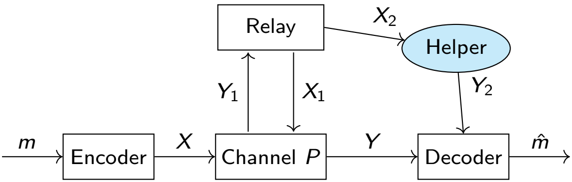
- Average rate with $k > 1$
- The continuous case, where

$$C \sim q, \quad \operatorname{supp} q = [0, C_{max}]$$

RC with a (reliable) helper

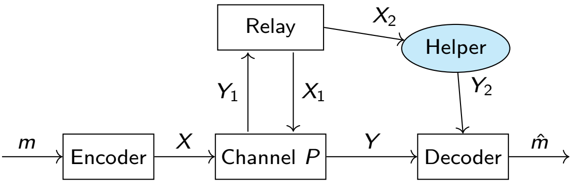


RC with a (reliable) helper



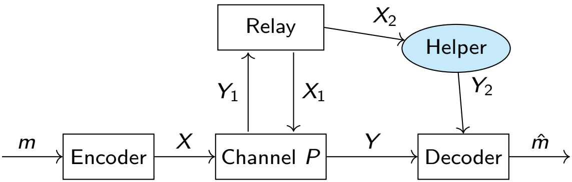
- The helper is either a conference link of a given capacity C_1, or a memoryless channel $P(y_2|x_2)$

RC with a (reliable) helper



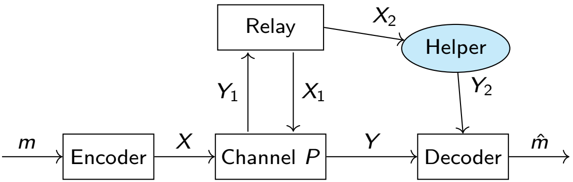
- The helper is either a conference link of a given capacity C_1, or a memoryless channel $P(y_2|x_2)$
- General model: $\tilde{P}(y, y_1, y_2|x, x_1, x_2) = p(y, y_1|x, x_1)p(y_2|x_2)$

RC with a (reliable) helper



- The helper is either a conference link of a given capacity C_1, or a memoryless channel $P(y_2|x_2)$
- General model: $\tilde{P}(y, y_1, y_2|x, x_1, x_2) = p(y, y_1|x, x_1)p(y_2|x_2)$
- Helper is decoupled from main channel

RC with a (reliable) helper



- The helper is either a conference link of a given capacity C_1, or a memoryless channel $P(y_2|x_2)$
- General model: $\tilde{P}(y, y_1, y_2|x, x_1, x_2) = p(y, y_1|x, x_1)p(y_2|x_2)$
- Helper is decoupled from main channel
  - RC with additional primitive element, or receiver orthogonal component

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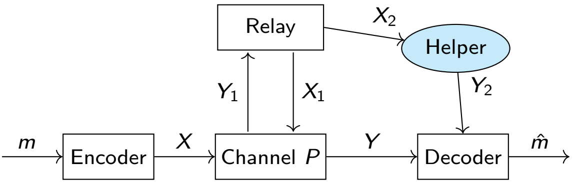

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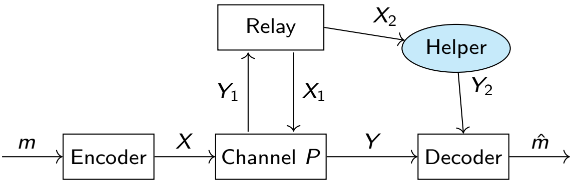

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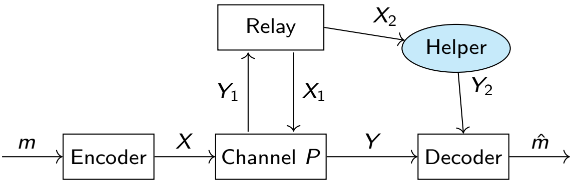
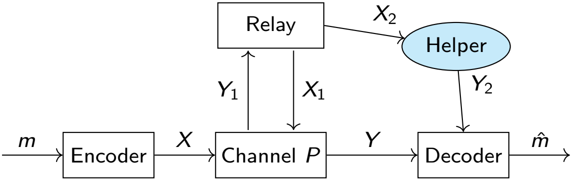
$$C = \max_{P_{X,X_1}} \min\{I(X, X_1; Y) + C_1, I(X; Y_1|X_1)\}$$

RC with a (reliable) helper



Is a decoupled helper a realistic model?

RC with a (reliable) helper



Is a decoupled helper a realistic model?

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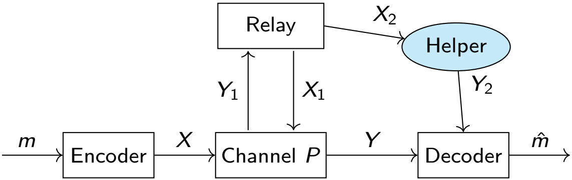

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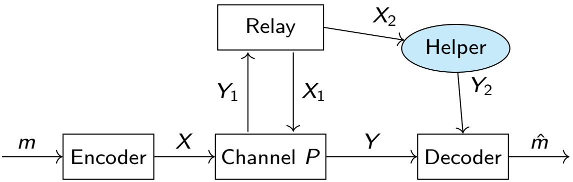

Is a decoupled helper a realistic model?

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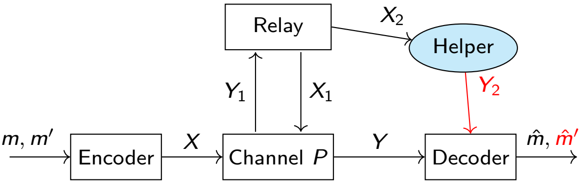
...but his presence is not guaranteed a priori.

RC with unreliable helper

Setup:



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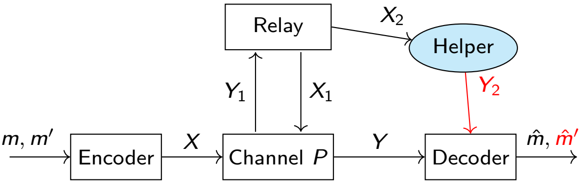

- The pair of messages (m, m') is always sent

RC with unreliable helper

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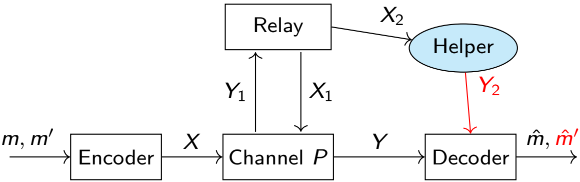

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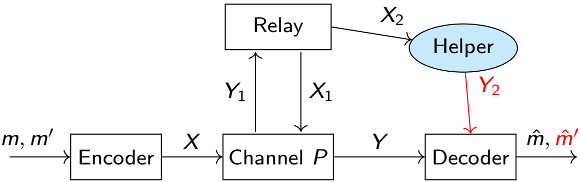

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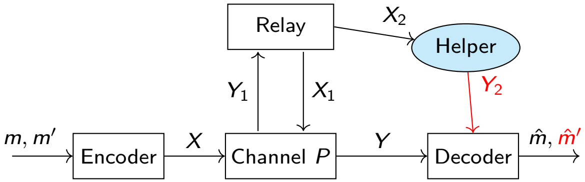

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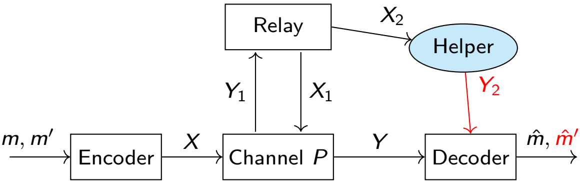

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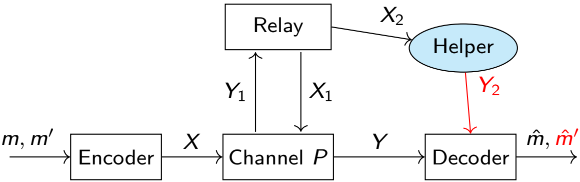
Characterize the set of achievable rates (R, R').

RC with unreliable helper

Related problems



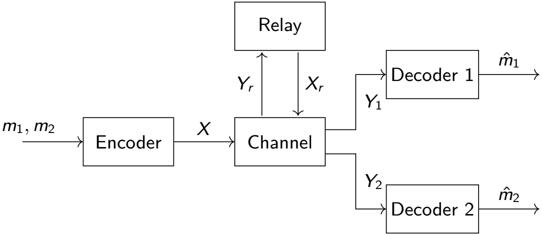
Closely related to

- Broadcast channel (BC) with unreliable conferencing [S. 2014, Itzhak & S. 2017, 2021]
- Relay Broadcast channels (RBC) [Liang 2005, Behboodi & Piantanida 2013, Hu *et. al.* 2020, Bhaskaran 2008]

RC with unreliable helper

Related problems - dedicated RBC

Liang & Veeravalli '07, Liang '05:



RC with unreliable helper

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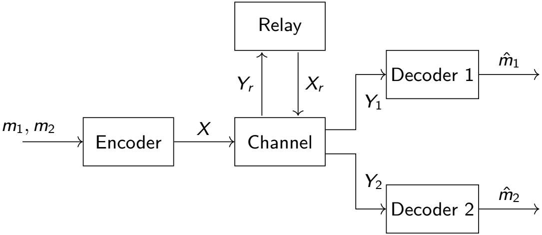

Solved for:

RC with unreliable helper

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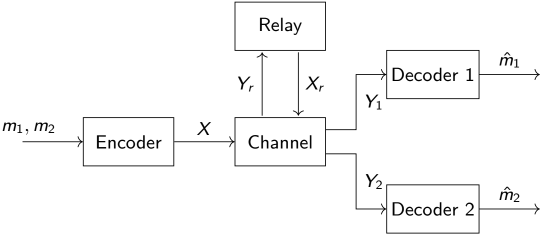

Solved for:

- Y_1 is reversely degraded and Y_2 is degraded w.r.t the relay [Liang '05, Behboodi & Piantanida '13]

RC with unreliable helper

Related problems - dedicated RBC

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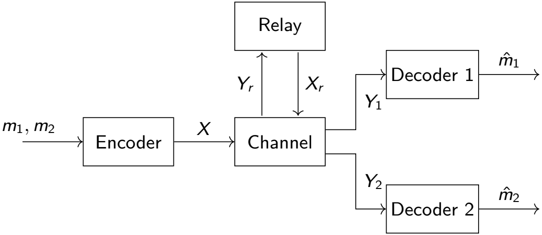

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- Both outputs are reversely degraded [Hu,Wang, Ma and Wu, '20]

RC with unreliable helper

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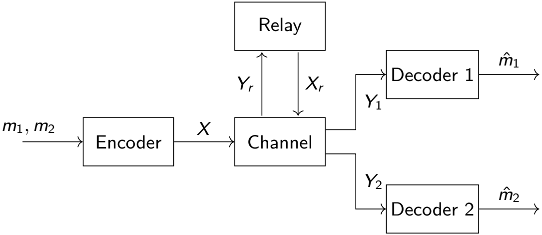

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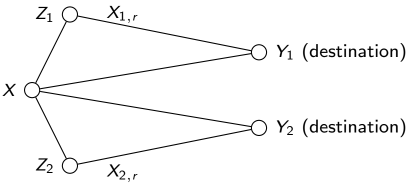
- Both outputs are reversely degraded [Hu,Wang, Ma and Wu, '20]
- Gaussian channels, both degraded [Bhaskaran '08]

RC with unreliable helper

Related problems - other forms of dedicated RBCs

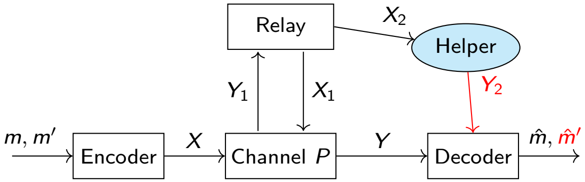
Two independent relays (= two RCs with common input)



Bounds derived by Behboodi & Piantanida '13.

RC with unreliable helper

Back to our problem:



Characterize the set of achievable rates (R, R').

Code

$\mathcal{N}$, $\mathcal{N}'$ - two message sets. A code is an encoder f

$$f:\ \mathcal{N} \times \mathcal{N}' \to \mathcal{X}^n,$$

a causal relay encoder $g = \{g_1, \ldots, g_n\}$

$$g_i : \mathcal{Y}_1^{i-1} \to \mathcal{X}_{1,i} \times \mathcal{X}_{2,i},$$

and a pair of decoders ϕ, ϕ':

$$\phi : \mathcal{Y}^n \to \mathcal{N}$$
$$\phi' : \mathcal{Y}^n \times \mathcal{Y}_2^n \to \mathcal{N} \times \mathcal{N}'$$

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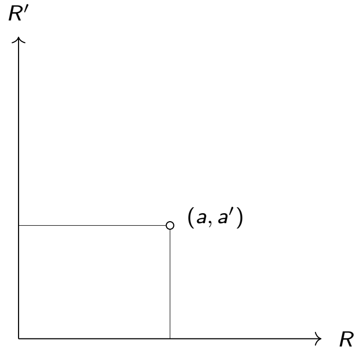
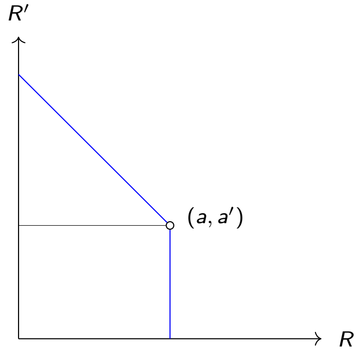
Observation: If (R, R') is achievable, so are pairs $(\tilde{R}, \tilde{R}')$ satisfying

$$\tilde{R} < R$$
$$\tilde{R} + \tilde{R}' < R + R'.$$


R'
(a, a')
R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RC with unreliable helper

The capacity region for the degraded case

[S. 23]
The capacity region of the degraded RC with unreliable helper is the collection of all pairs (R, R') satisfying

$$
\begin{aligned}
R &\leq I(U, X_1; Y) \\
R + R' &\leq \min \{ I(X, X_1; Y) + C_1, I(X; Y_1|X_1), \\
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for some $P(u, x, x_1)$ such that

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and C_1 is the capacity of the helper channel, i.e.,

$$C_1 = \max_{P_{X_2}} I(X_2; Y_2).$$

Degraded RC with unreliable helper

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(it is the total rate of the superposition coding)

Degraded RC with unreliable helper

Method of proof

Direct part:

Degraded RC with unreliable helper

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Direct part: Superposition, block Markov and binning, with:

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Degraded RC with unreliable helper

Method of proof

Direct part: Superposition, block Markov and binning, with:

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Degraded RC with unreliable helper

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Direct part: Superposition, block Markov and binning, with:

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Degraded RC with unreliable helper

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Degraded RC with unreliable helper

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Converse: Chain rule for I, convexity...

Degraded RC with unreliable helper

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Superposition coding + block Markov, d&f and binning yield the achievability result:

Degraded RC with unreliable helper

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Superposition coding + block Markov, d&f and binning yield the achievability result:

$$R < I(U, X_1; Y) \tag{7}$$

$$R' < \min\{I(X; Y|UX_1) + C_1, I(X; Y_1|UX_1)\} \tag{8}$$

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Degraded RC with unreliable helper

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Now use Observation 1, and replace (8) by the sum of (7) and (8).

Degraded RC with unreliable helper - discussion
Special cases

1. $C_1 = 0$. We obtain

$$R + R' = \max_{P(x,x_1)} \min\{I(X, X_1; Y), I(X; Y_1|X_1)\}$$

Degraded RC with unreliable helper - discussion

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Degraded RC with unreliable helper - discussion

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2. $R = 0$, or don't care. Here we maximize the rate with the helper.

$$R' = \max_{P(x,x_1)} \min\{I(X, X_1; Y) + C_1, I(X; Y_1|X_1)\}$$

3. $R' = 0$. Here we maximize the rate without the helper.

Degraded RC with unreliable helper - discussion

Special cases

1. $C_1 = 0$. We obtain

$$R + R' = \max_{P(x,x_1)} \min\{I(X, X_1; Y), I(X; Y_1|X_1)\}$$

2. $R = 0$, or don't care. Here we maximize the rate with the helper.

$$R' = \max_{P(x,x_1)} \min\{I(X, X_1; Y) + C_1, I(X; Y_1|X_1)\}$$

3. $R' = 0$. Here we maximize the rate without the helper. We get

$$R = \max_{P(x,x_1)} \min\{I(X, X_1; Y), I(X; Y_1|X_1)\},$$

like case 1 above.

Degraded RC with unreliable helper - discussion

The role of U, $U \multimap (X, X_1) \multimap (Y, Y_1)$

1. $U = X$ leads to the capacity of the channel without helper

Degraded RC with unreliable helper - discussion

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1. $U = X$ leads to the capacity of the channel without helper
2. $U = X_1$.

Degraded RC with unreliable helper - discussion

The role of U, $U \multimap (X, X_1) \multimap (Y, Y_1)$

1. $U = X$ leads to the capacity of the channel without helper
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$$R < I(X_1; Y) \tag{10}$$

$$R + R' < \min\{I(X, X_1; Y) + C_1, I(X; Y_1|X_1)\} \tag{11}$$

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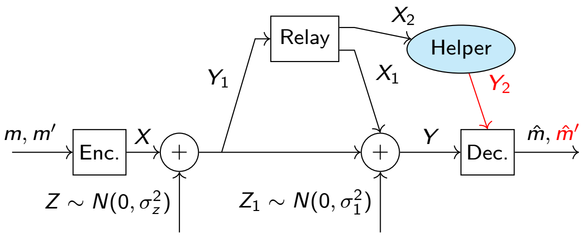
3. U null random variable leads to case 2 above.

Degraded RC with unreliable helper

Gaussian degraded RC



$$Y_1 = X + Z$$
$$Y = Y_1 + X_1 + Z_1 = X + X_1 + Z + Z_1,$$

with

$$\mathbb{E}(X^2) \leq P, \quad \mathbb{E}(X_1^2) \leq P_1.$$

Gaussian degraded RC with unreliable helper

The capacity region

The capacity region is the collection of all pairs (R, R') satisfying

$$R \leq \mathcal{C}\left(\frac{\overline{\alpha\beta}P + P_1 + 2\sqrt{\bar{\alpha}PP_1}}{\alpha\beta P + \sigma_z^2 + \sigma_1^2}\right) \tag{12}$$

$$R + R' \leq \min\left\{\mathcal{C}\left(\frac{P + P_1 + 2\sqrt{\bar{\alpha}PP_1}}{\sigma_z^2 + \sigma_1^2}\right) + C_1, \mathcal{C}\left(\frac{\alpha P}{\sigma_z^2}\right),\right.$$
$$\left.\mathcal{C}\left(\frac{\overline{\alpha\beta}P + P_1 + 2\sqrt{\bar{\alpha}PP_1}}{\alpha\beta P + \sigma_z^2 + \sigma_1^2}\right) + \mathcal{C}\left(\frac{\alpha\beta P}{\sigma_z^2}\right)\right\} \tag{13}$$

for some $\alpha, \beta \in [0, 1]$, where $\bar{\alpha} = 1 - \alpha$.

Gaussian degraded RC with unreliable helper

Capacity results

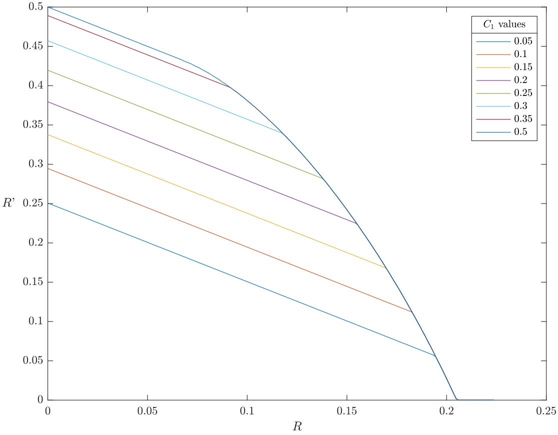
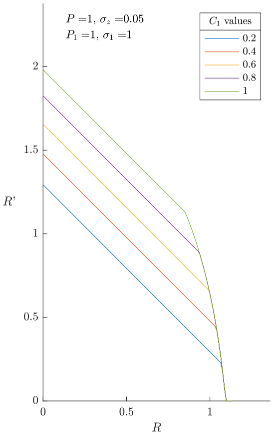
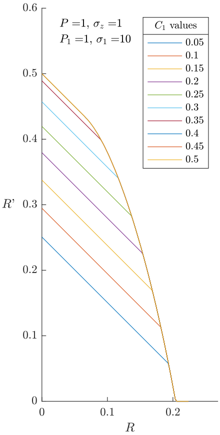


Figure: The capacity region of the Gaussian degraded relay channel with unreliable helper, for $P = P_1 = \sigma_z^2 = 1$, $\sigma_1^2 = 10$.

Gaussian degraded RC with unreliable helper

Capacity results

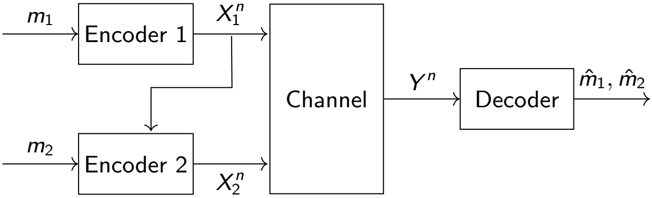




The MAC with cribbing encoders

Reliable cribbing

(W & VDM 85)



Single sided strictly causal cribbing model:

$$x_{1,i} = f_{1,i}(m_1)$$
$$x_{2,i} = f_{2,i}(m_2, x_1^{i-1})$$

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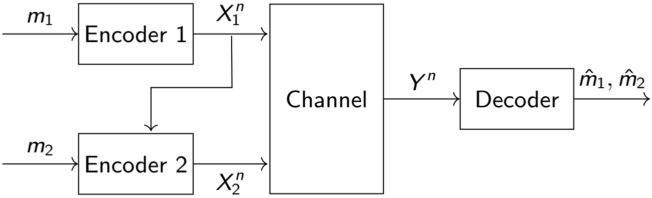

Single sided strictly causal cribbing model, apacity region

$$
\begin{aligned}
R_1 &\leq H(X_1|U) \\
R_2 &\leq I(X_2; Y|X_1, U) \\
R_1 + R_2 &\leq I(X_1, X_2; Y)
\end{aligned}
$$

for some $P_U P_{X_1|U}, P_{X_2|U} P_{Y|X_1,X_2}$.

The MAC with cribbing encoders

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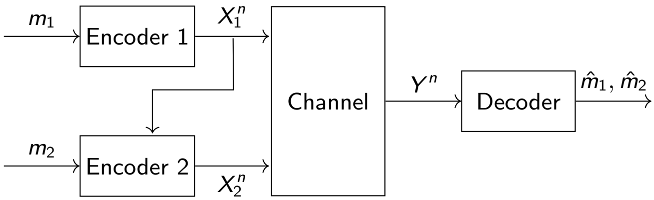

Single sided causal cribbing model:

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The MAC with cribbing encoders

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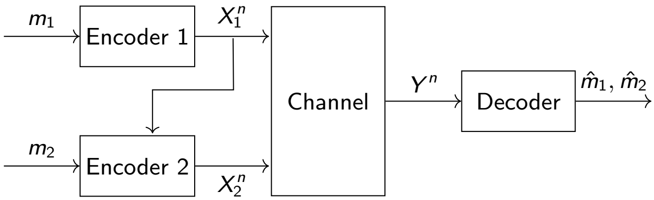


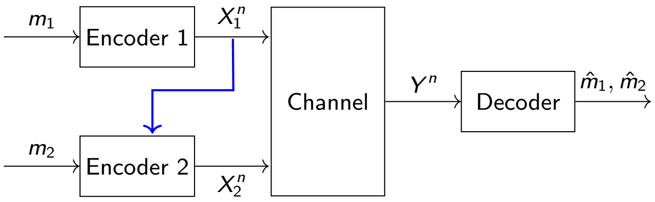
Single sided causal cribbing model, capacity region:

$$\begin{aligned} R_1 &\leq H(X_1) \\ R_2 &\leq I(X_2; Y|X_1) \\ R_1 + R_2 &\leq I(X_1, X_2; Y) \end{aligned}$$

for some $P_{X_1,X_2} P_{Y|X_1,X_2}$.

MAC with unreliable cribbing

(S. 2014, Huleihel & S. 16, 17)



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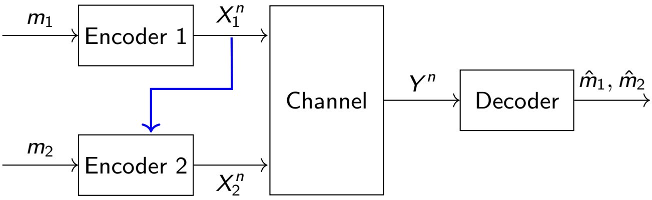
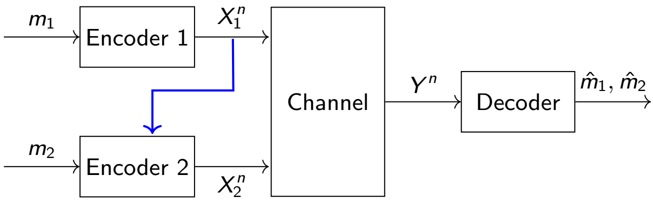


- User 1 does not know whether cribbing link is present. He is only aware that cribbing could occur.
- User 2 and the decoder know whether or not the cribbing link is present

MAC with unreliable cribbing

(S. 2014, Huleihel & S. 16, 17)



- R_1 - rate of User 1. Always decoded.

MAC with unreliable cribbing

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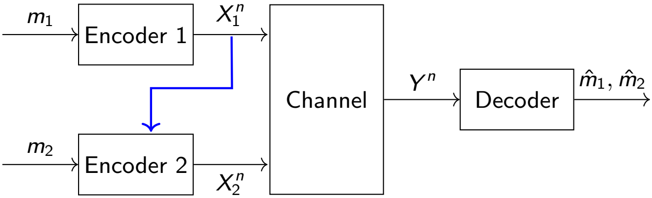

- R_1 - rate of User 1. Always decoded.
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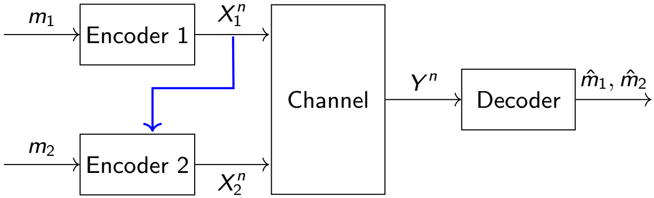

- R_1 - rate of User 1. Always decoded.
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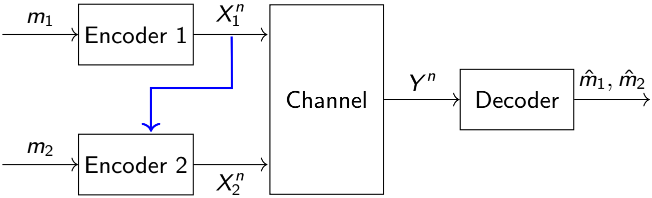
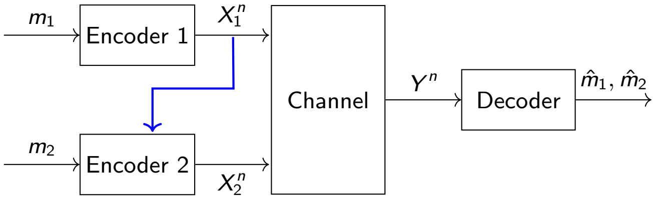


- R_1 - rate of User 1. Always decoded.
- R_1' - extra rate of User 1, decoded only if User 2 cribs
- R_2 - rate of User 2. Decoded only if he does not crib.
- R_2'' - rate of User 2. Decoded only if he cribs (can be lower than R_2).

MAC with unreliable cribbing

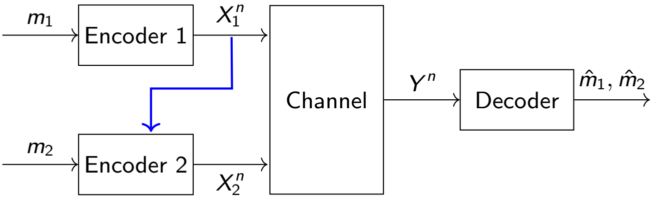


An achievable region, strictly causal cribbing:

$$R_1 \leq I(V; Y|X_2)$$
$$R_2 \leq I(X_2; Y|V)$$
$$R_1 + R_2 \leq I(V, X_2; Y)$$

for some $P_U P_V P_{X_1|U,V} P_{X_2} P_{Y|X_1,X_2}$

MAC with unreliable cribbing



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R_1' + R_2'' &\leq I(X_1, X_2''; Y''|V) \\
R_1 + R_1' + R_2'' &\leq I(X_1, X_2''; Y'')
\end{aligned}
$$

for some $P_U P_V P_{X_1|U,V} P_{X_2} P_{Y|X_1,X_2} P_{X_2''|U} P_{Y|X_1,X_2''}$.

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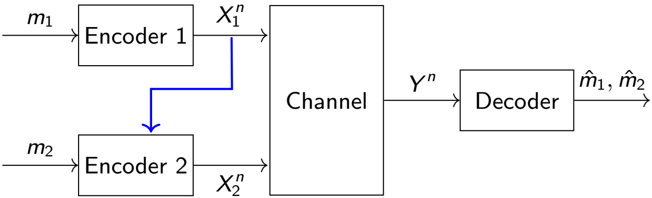

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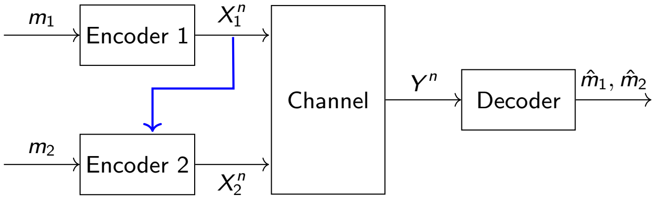

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MAC with unreliable cribbing

Outer bound - causal and non-causal cribbing:

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- Many degrees of uncertainly solved for the BC
- An average rate criteria, suitable for many degrees of uncertainty, is suggested, and studied for the BC. Continuum uncertainty for Gaussian BC is solved.
- MAC with unreliable cribbing was suggested, and bounds derived.

Future work

- RC: many degrees of uncertainty, average rate criterion, continuous uncertainty

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- Other channel models. Gaussian MIMO channels.

Thank You!