Physical Unclonable Functions

Coded Modulation, Shaping, and Helper Data Schemes

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Acknowledgment

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Introduction

Introduction

Physical Unclonable Functions (PUFs): [PRTG'02], [GCDD'02], [SN'00], [BH'13], [Mae'13]

- physical hardware object
- unique, unpredictable, and uncontrollable
  due to random physical processes at the time of production
- cannot be duplicated or cloned, i.e., are physically unclonable

Modes of Operation:

- "strong" PUFs: the response is dependent on a challenge
- "weak" PUFs: a unique fingerprint is delivered (considered here)
  maybe better: *physical unclonable "object" / physical unclonable "fingerprint"*

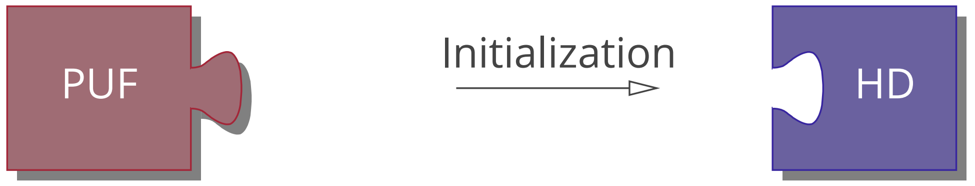
Observation and Approach:

- repeated PUF readout vary (slightly)
  due to variations in temperature, supply voltage, aging, . . .
- readout has to be stabilized — channel coding has to be applied

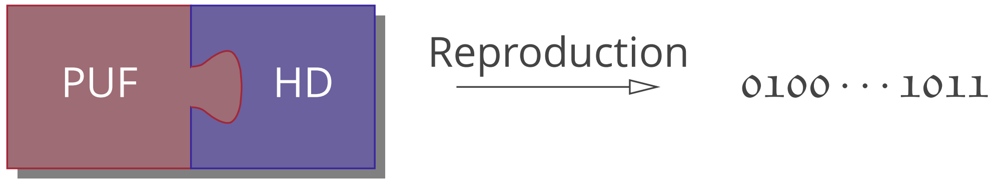
Introduction (II)

Procedure: fuzzy extractors / secure sketch [DRS'07]

- ***Initialization / Enrollment***
  based on the PUF readout ***helper data (HD) is generated***
  the helper data must not reveal any information about the PUF readout and may be public



- ***Reproduction***
  based on the noisy PUF readout and the helper data a stable (binary) word / key is generated



Applications:

- derivation of cryptographic keys / inherent key storage
  the PUF is private and the helper data may be public
- identification / countermeasure against product piracy
  the PUF is public and the helper data is private

Introduction (III)

Research Areas and Directions:

Microelectronics	***Computer Science***
more stable PUF architectures, efficient implementation of coding schemes, . . .	protocols, security, attacks, . . .
e.g., [MHV'12], [HBO'16], [MHK+'19], [KFPW'22]	e.g., [GCDD'02], [DRS'07], [MSSS'12], [Teb'22]
Information Theory	***Coding Theory***
fundamental procedures and limits, . . .	design suited channel coding schemes
e.g., [AC'93], [Mau'93], [CN'00], [IW'09], [GFBP'23]	e.g., [Mae'13], [PMBHS'15], [Müe'19], [FM'22]

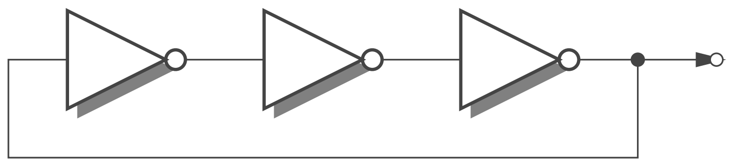
Overview

- *Classical Binary PUFs and Problem Statement*
- *Soft-Output PUFs*
- *Coded Modulation and Shaping*
- *Helper Data for Improved Decoding*
- *FPGA Implementation*

Classical Binary PUFs

Ring Oscillator PUFs

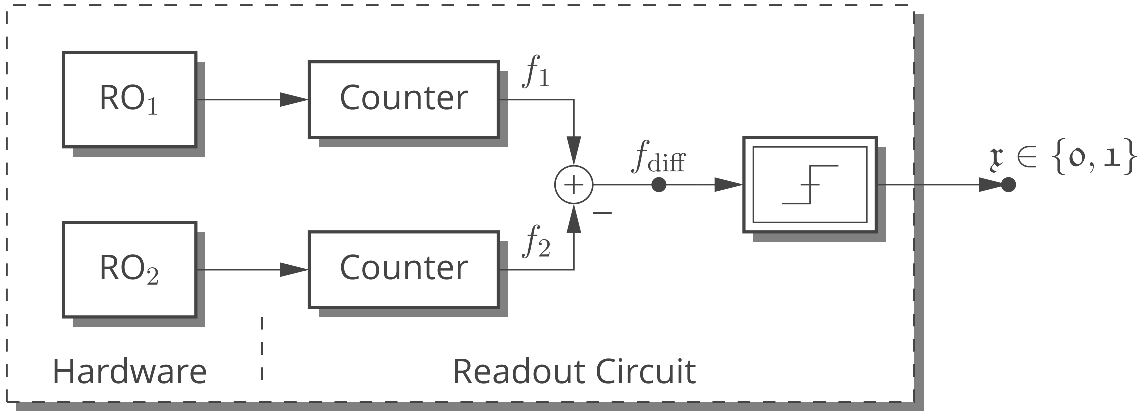
Ring Oscillator: ("silicon PUF")



- loop of an odd number of inverters (NOT gates)
- the circuit oscillates with a certain frequency
  actual value depends on uncontrollable variations within the manufacturing process

Classical Ring Oscillator PUF (ROPUF): [GCDD'02]

- sign of frequency difference f_{diff} is extracted



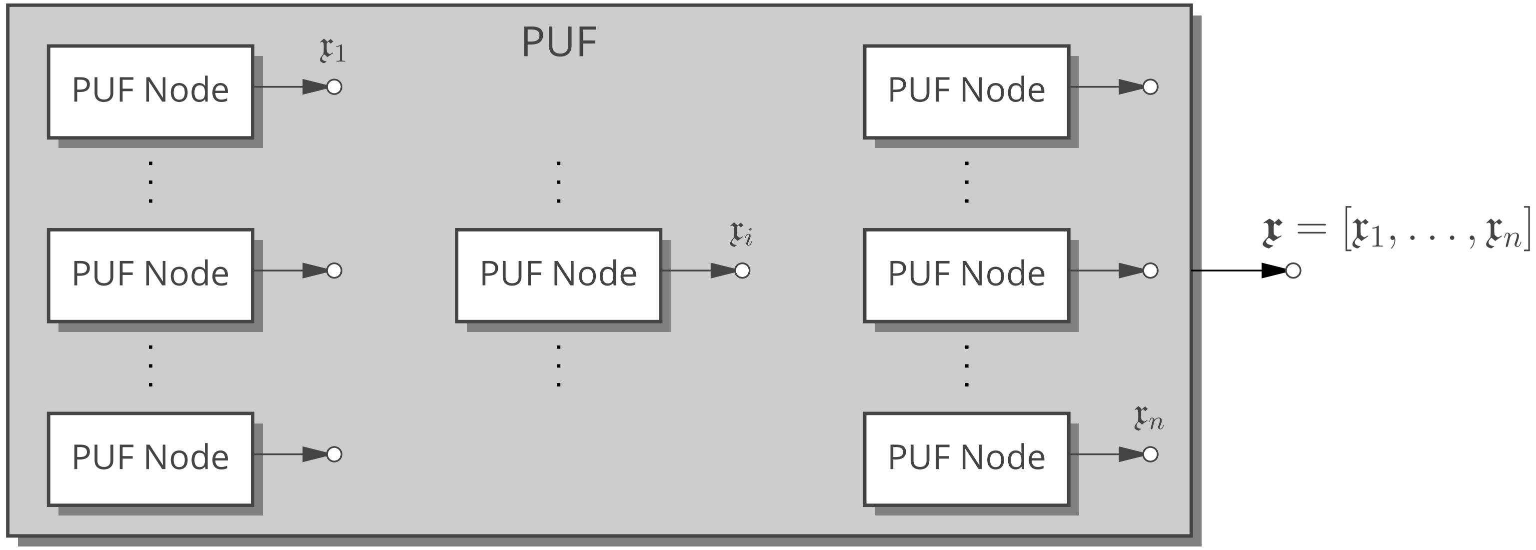
- basic block for generating a single random variable — ***PUF node***, ***PUF cell***, or ***PUF unit***

Notation: quantities over $\mathbb{R}$ are typeset as $\boldsymbol{x}$, $\boldsymbol{e}$, … — quantities over $\mathbb{F}_2$ are typeset in Fraktur font; $\mathfrak{x}$, $\boldsymbol{\mathfrak{x}}$, …

Classical PUFs

Extracted Information / Entire PUF:

- n ***independent*** PUF nodes constitute the PUF



- PUF readout vector $\mathfrak{x} = [\mathfrak{x}_1, \ldots, \mathfrak{x}_n] \in \mathbb{F}_2^n$
- $\mathfrak{x}_i$ ***uniformly*** and ***independently*** distributed
- each PUF instance has a unique readout $\mathfrak{x}$

Classical PUFs (II)

Extracted Information:

- each PUF instance has a unique *reference readout* $\mathfrak{x}_{\text{puf}}$ ⇒ *randomness in the manufacturing process*

$$\mathfrak{x}_{\text{puf},1} \; \mathfrak{x}_{\text{puf},2} \; \mathfrak{x}_{\text{puf},3} \; \cdots\cdot \; \mathfrak{x}_{\text{puf},i} \; \cdots\cdot \; \mathfrak{x}_{\text{puf},n}$$

Problem:

- repeatedly requested readouts will vary (slightly) ⇒ *randomness in the readout process*
  due to variations in temperature, supply voltage, aging, ...

- instability is traditionally modeled by a binary symmetric channel (BSC)

$$\boldsymbol{\mathfrak{y}}_{\text{puf}} = \boldsymbol{\mathfrak{x}}_{\text{puf}} \oplus \boldsymbol{\mathfrak{e}}_{\text{puf}}$$

  error pattern $\boldsymbol{\mathfrak{e}}_{\text{puf}}$ — usual assumption: bit error probability $p_{\text{BSC}} \approx 0.14$ e.g., [MHV'12], [MPMHS'14], [PMBHS'15]

  ⇒ *employ channel coding*

- However: the reference PUF readout $\boldsymbol{\mathfrak{x}}_{\text{puf}}$ is not a valid code word

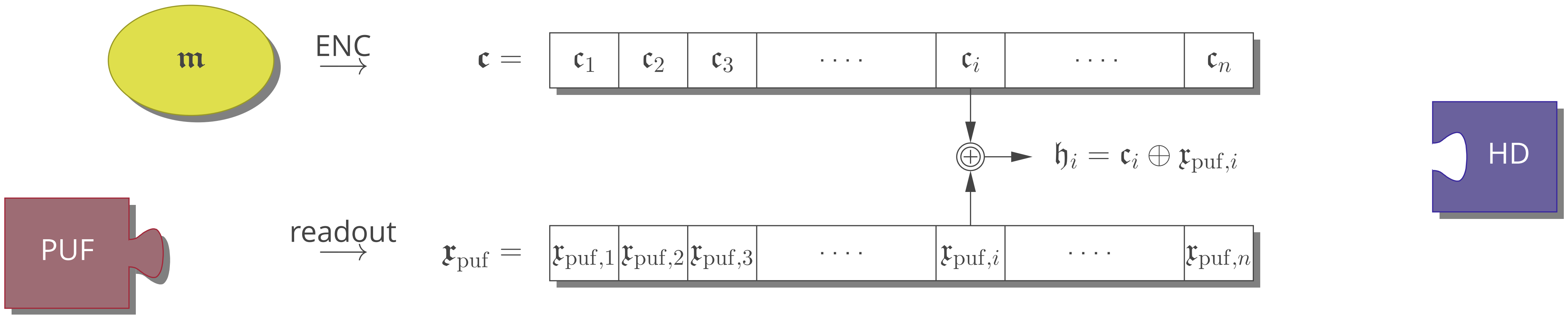
Classical PUFs (III)

Initialization / Enrollment Phase:

- the ***reference PUF readout*** $\mathfrak{x}_{\text{puf}}$ is measured
- choice: – binary channel code (rate k/n)
  – binary message word $\mathfrak{m}$ of length k — the corresponding code word $\mathfrak{c}$ is generated
- ***helper data*** is calculated as — ***code-offset algorithm*** e.g., [JW'99], [LT'03], [DRS'04]

$$\mathfrak{h} \overset{\text{def}}{=} \mathfrak{c} \oplus \mathfrak{x}_{\text{puf}}$$

- visualization



Reproduction Phase:

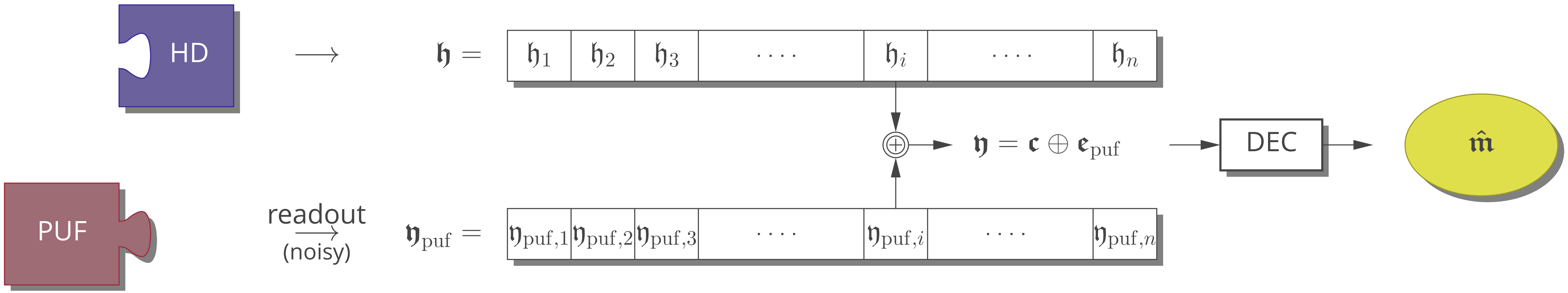
- noisy PUF readout

$$\boldsymbol{\mathfrak{y}}_{\text{puf}} = \boldsymbol{\mathfrak{x}}_{\text{puf}} \oplus \boldsymbol{\mathfrak{e}}_{\text{puf}} = \boldsymbol{\mathfrak{c}} \oplus \boldsymbol{\mathfrak{h}} \oplus \boldsymbol{\mathfrak{e}}_{\text{puf}}$$

- application of helper data

$$\boldsymbol{\mathfrak{y}} \stackrel{\text{def}}{=} \boldsymbol{\mathfrak{y}}_{\text{puf}} \oplus \boldsymbol{\mathfrak{h}} = \boldsymbol{\mathfrak{c}} \oplus \boldsymbol{\mathfrak{e}}_{\text{puf}}$$

- visualization

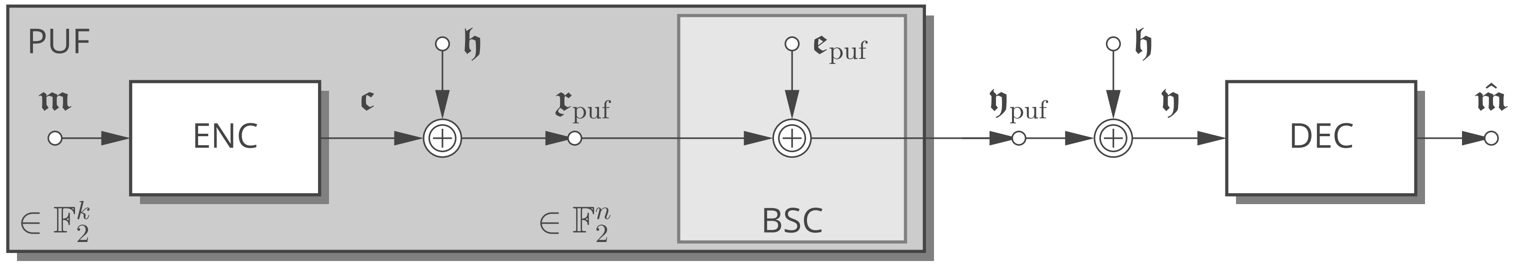


- standard (hard-decision) channel decoding reveals the message $\boldsymbol{\mathfrak{m}}$

Classical PUFs (V)

Model of the PUF:

- visualization



- randomness in the manufacturing process — $\mathfrak{x}_{\mathrm{puf}}$
- randomness in the readout process — $\mathfrak{e}_{\mathrm{puf}}$

Imagination of a Digital Communication Scheme:

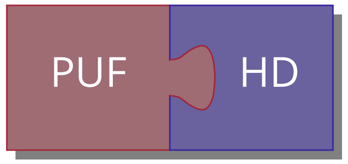
- randomly selected message $\mathfrak{m}$ of length k
- encoding and application of helper data gives $\mathfrak{x}_{\mathrm{puf}}$
- secret (key) to be retrieved: *message* $\mathfrak{m}$

Security of PUFs

Requirements: ($\mathrm{I}(\cdot;\ \cdot)$: mutual information)

- the PUF (reference) readout $\mathfrak{x}_{\text{puf}}$ and the helper data $\mathfrak{h}$ are known

  ⇒ *the message $\mathfrak{m}$ has to be decodable*



$$\mathrm{I}(\mathfrak{m};\ \{\mathfrak{x}_{\text{puf}}\ , \mathfrak{h}\}) = k$$

- only the PUF (reference) readout $\mathfrak{x}_{\text{puf}}$ is known

  ⇒ *no leakage must occur*

$$\mathrm{I}(\mathfrak{m};\ \mathfrak{x}_{\text{puf}}) = 0$$

- only the helper data $\mathfrak{h}$ is known

  ⇒ *no leakage must occur*

$$\mathrm{I}(\mathfrak{m};\ \mathfrak{h}) = 0$$

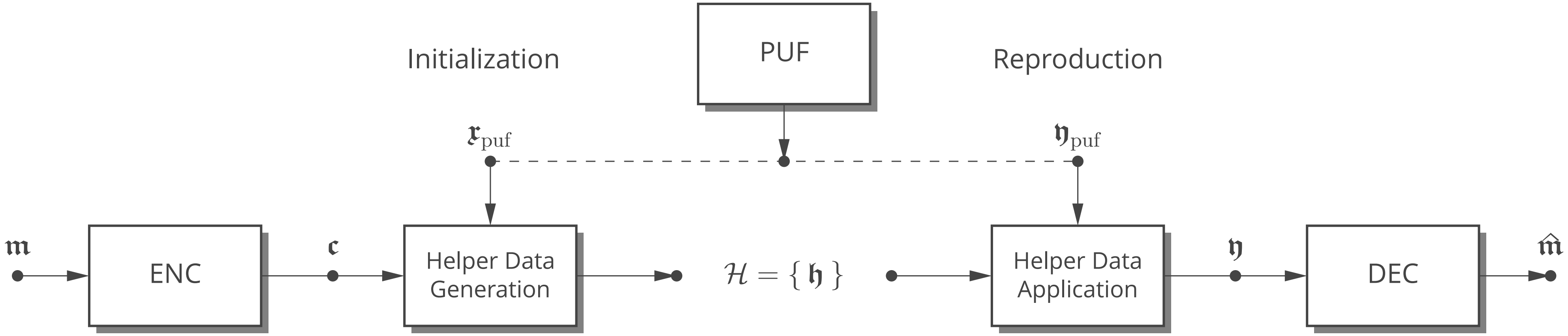
Interpretation:

- the readout $\mathfrak{x}_{\text{puf}}$ is a *one-time pad* for the codeword $\mathfrak{c}$ and vice versa

Interpretation

Channel Coding Problem:

- generation of and communication via ***helper data***

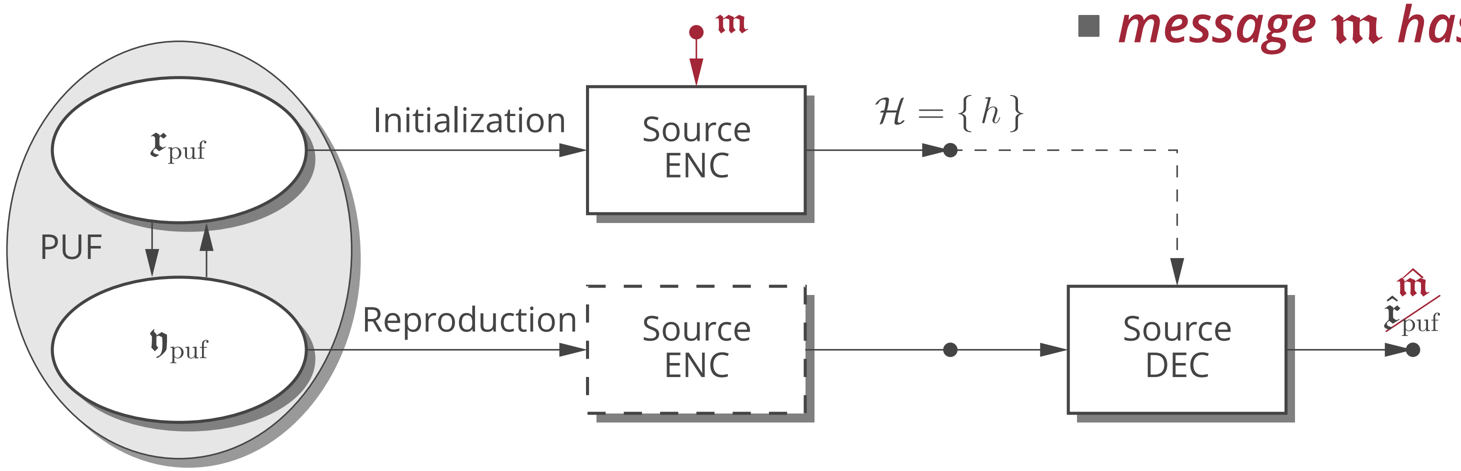


Source Coding Problem:

e.g., [GISK'19]

- Slepian–Wolf / Wyner–Ziv encoding

- ***message $\mathfrak{m}$ as additional randomness***
- ***message $\mathfrak{m}$ has to be recovered***



Numerical Examples

Word Error Ratio (WER) over the BSC Error Probability:

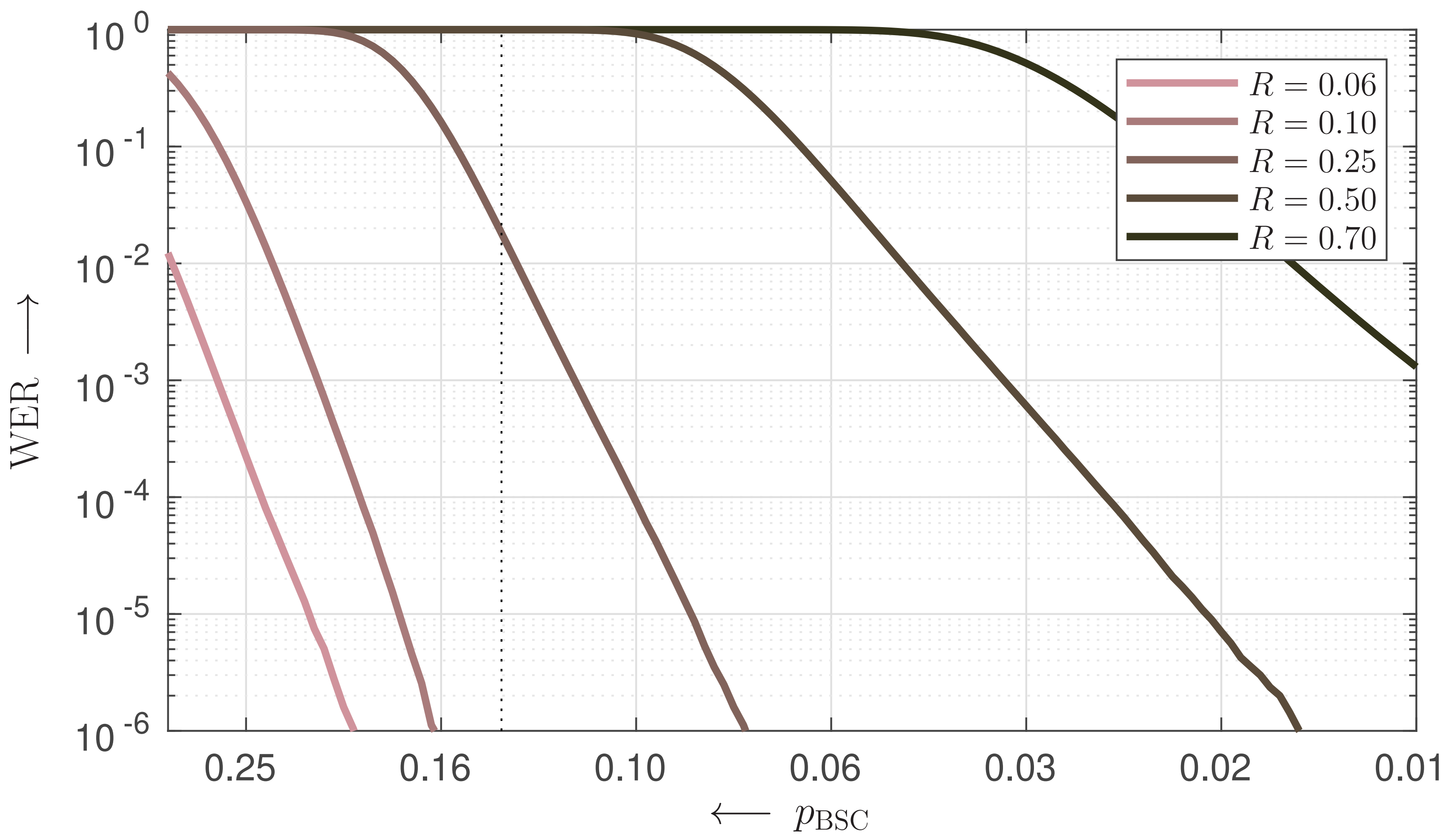
- PUF nodes: 1024

  mess. length: 61 to 717

  rate: $R = 0.06$ to 0.7 $\left[\frac{\text{bit}}{\text{node}}\right]$

- hard-decision decoding

- Polar code
  - codelength $n = 1024$
  - rate $R = 0.06$ to 0.7



Problem Statement

Channel Coding:

- ***Situation:*** vast majority of the literature is on ***binary*** codes and ***hard-decision*** decoding
- ***However:*** PUFs extract randomness from analog sources

Improvements: (the number n of PUF nodes is fixed)

- longer messages
  extract more than one bit of entropy per readout symbol $(k > n)$

  e.g., [TSB+'06], [BNCF'14], [GI'14], [WHGS'16] [ZPK+'16], [CBD+'17], [IOK+'18], [MHM+20]

  ⇒ ***multi-valued PUFs / coded modulation***

- higher reliability

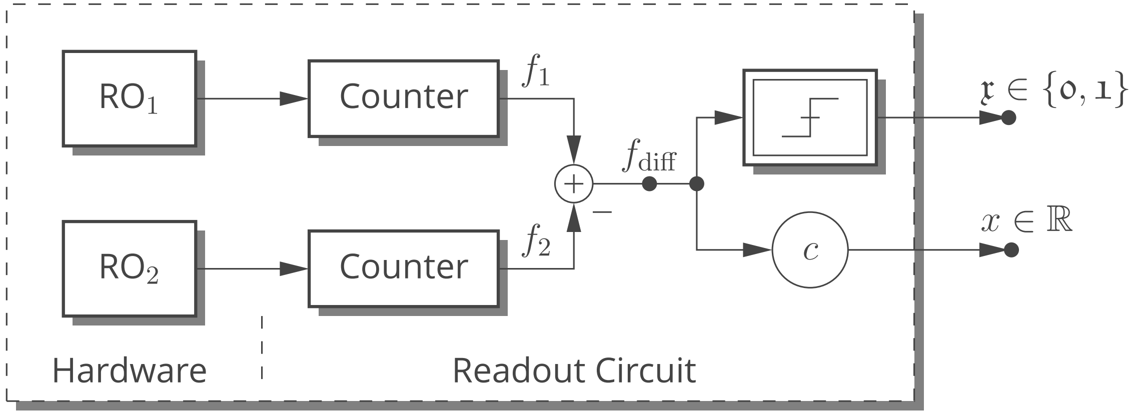
  e.g., [MTV'09], [MPSB'19], [MMOF'21], [KFPW'22]

  ⇒ ***utilize the soft output / advanced helper schemes***

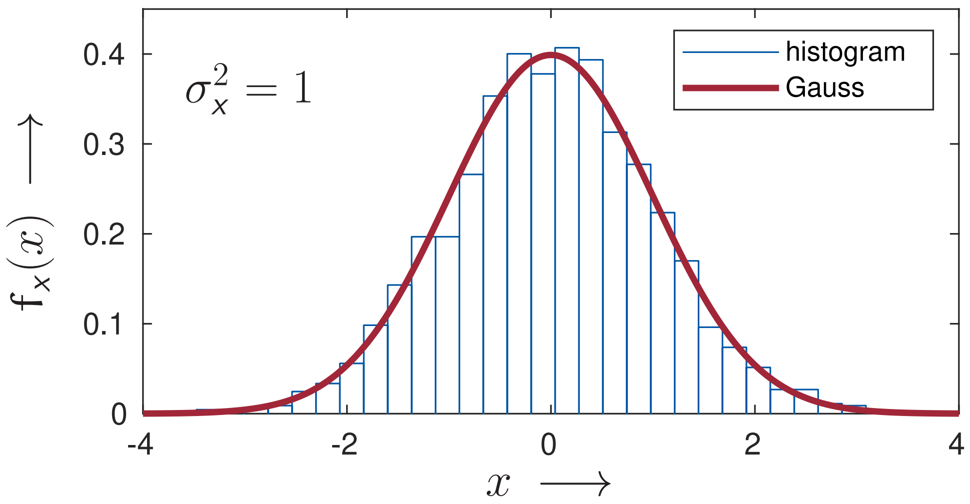
Soft-Output PUFs

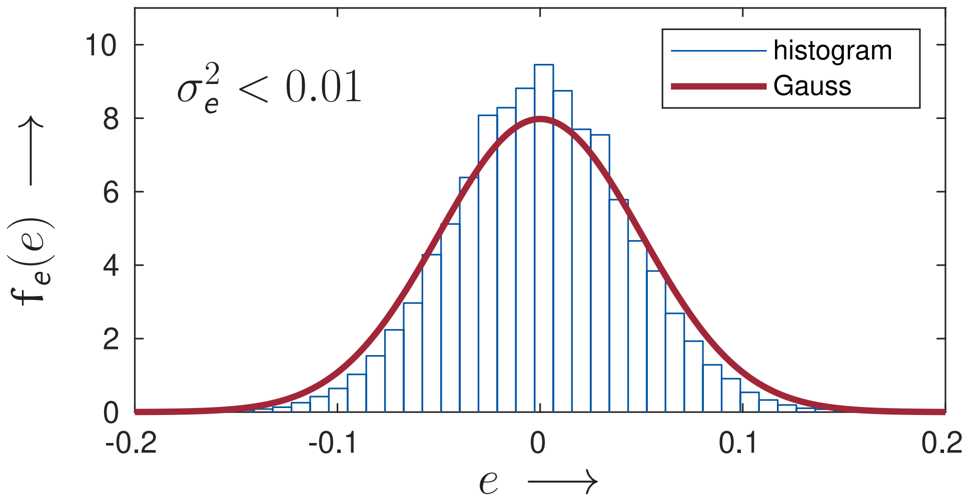
Soft-Decision Decoding:

- the real-valued frequency difference f_{diff} is utilized directly — reliability information



- measurement campaign at the Institute of Microelectronics using FPGA ROPUFs





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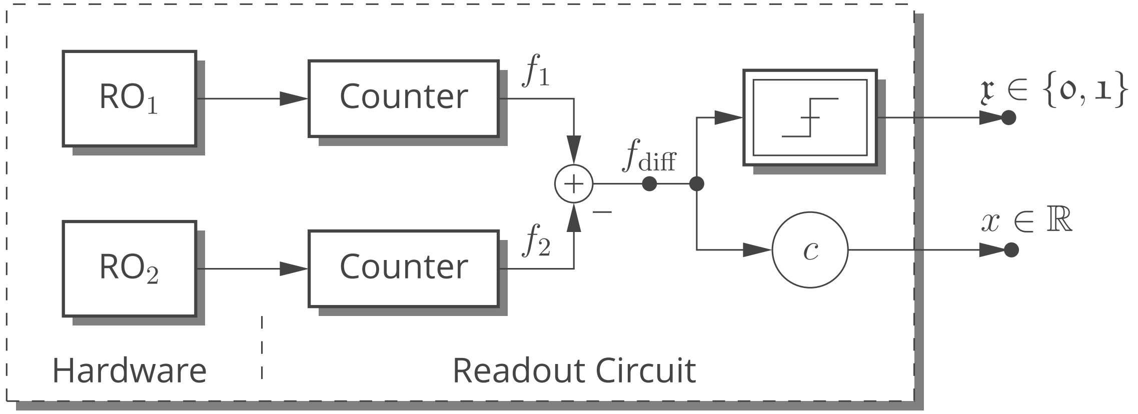


- AWGN model

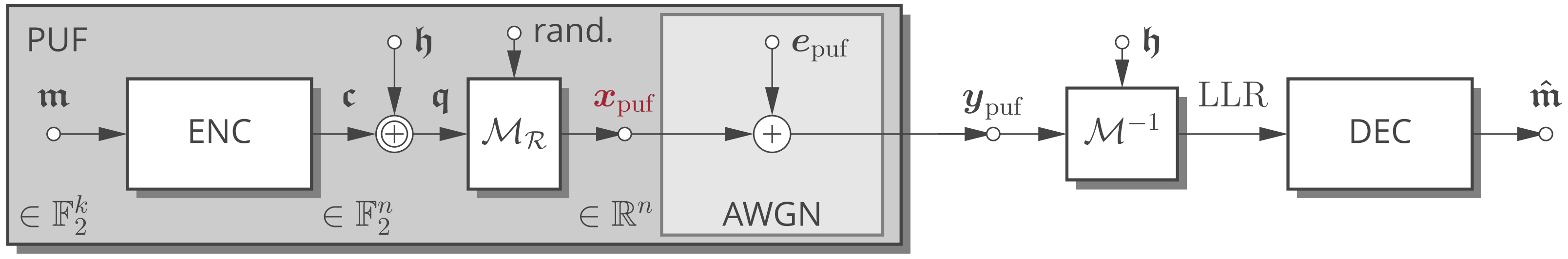
$$\boldsymbol{y}_{\mathrm{puf}} = \boldsymbol{x}_{\mathrm{puf}} + \boldsymbol{e}_{\mathrm{puf}}$$

  - reference/nominal readout $\boldsymbol{x}_{\mathrm{puf}}$ and error $\boldsymbol{e}_{\mathrm{puf}}$ are approx. zero-mean Gaussian distributed
  - scaling factor c such that $\sigma_x^2 = 1$ (per element)
  - error variance: $\sigma_e^2 < 0.01$

Soft-Output PUFs

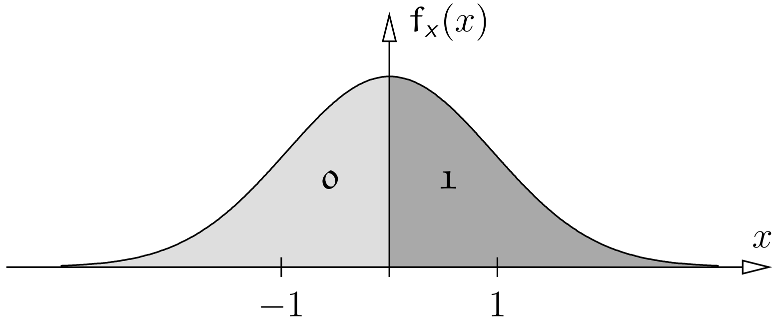
Model of the PUF:

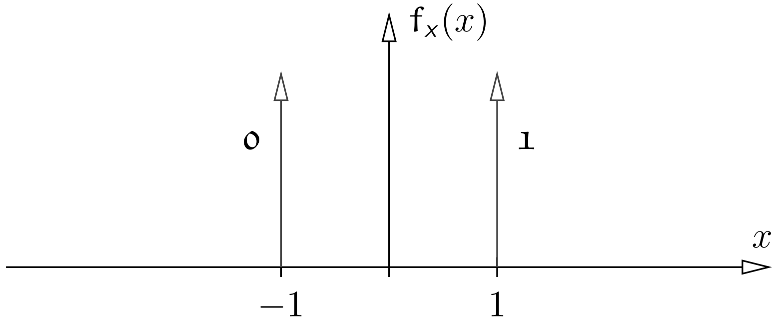
- we *imagine* a digital communication scheme — soft-decision



- random mapping — ***mapping bits to regions***
  - randomness at the transmitter
  - $\mathfrak{q}_i$ determines the region — the actual number $x_{\mathrm{puf},i}$ is drawn randomly according to a Gaussian pdf
  - individual but fixed for each PUF node (instance and position i in the codeword)

- Gaussian PUF readout vs. BPSK signaling





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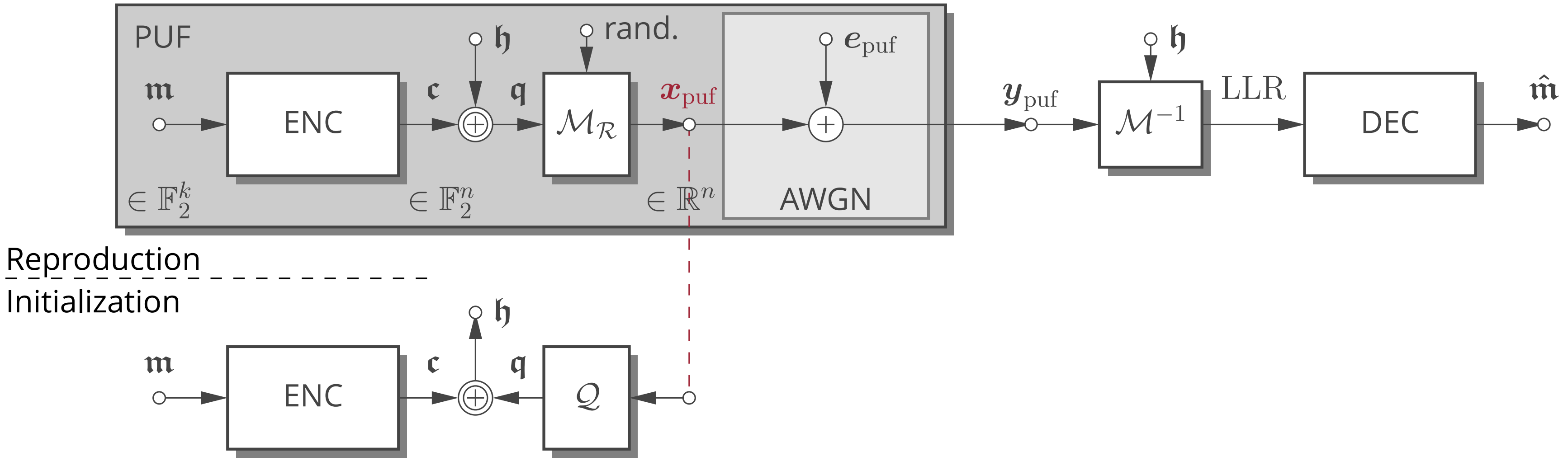


Initialization:

- determination of the actual region $\mathfrak{q}$
- encoding of the message to $\mathfrak{c}$
- calculation of helper data
  - $\mathfrak{c}$: desired region
  - $\mathfrak{q}$: actual region

$$\Rightarrow \quad \mathfrak{h} = \mathfrak{c} \oplus \mathfrak{q}$$

Soft-Output PUFs (II)

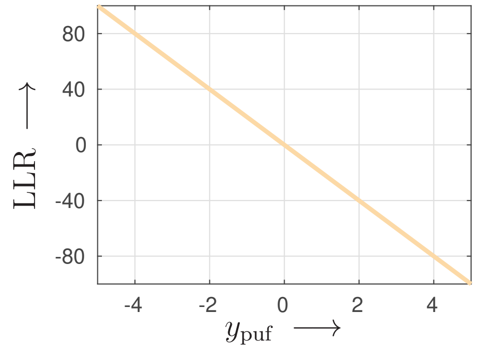
Soft-Decision Decoding:

- decoding metric: *log-likelihood ratio* (LLR)

$$\mathrm{LLR} = \log\left(\frac{\Pr\{\mathfrak{c}=0 \mid y_{\text{puf}}\}}{\Pr\{\mathfrak{c}=1 \mid y_{\text{puf}}\}}\right) = \log\left(\frac{f_y(y_{\text{puf}} \mid \mathfrak{c}=0)}{f_y(y_{\text{puf}} \mid \mathfrak{c}=1)}\right)$$

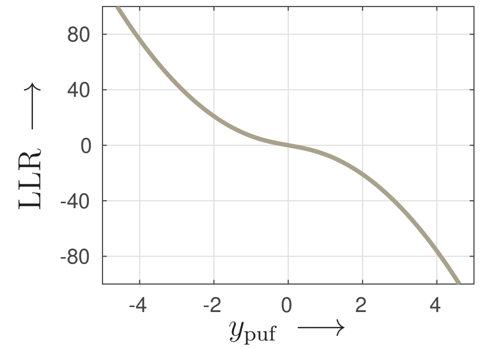
- BPSK over the AWGN channel

$$\mathrm{LLR} = \mp \frac{2}{\sigma_e^2}\, y_{\text{puf}}\,, \qquad \begin{array}{l} -,\ \mathfrak{h}=0 \\ +,\ \mathfrak{h}=1 \end{array}$$



- Gaussian PUF readout

$$\mathrm{LLR} = \pm \log\left(\frac{Q(+F\, y_{\text{puf}})}{Q(-F\, y_{\text{puf}})}\right)\,, \qquad \begin{array}{l} +,\ \mathfrak{h}=0 \\ -,\ \mathfrak{h}=1 \end{array}$$



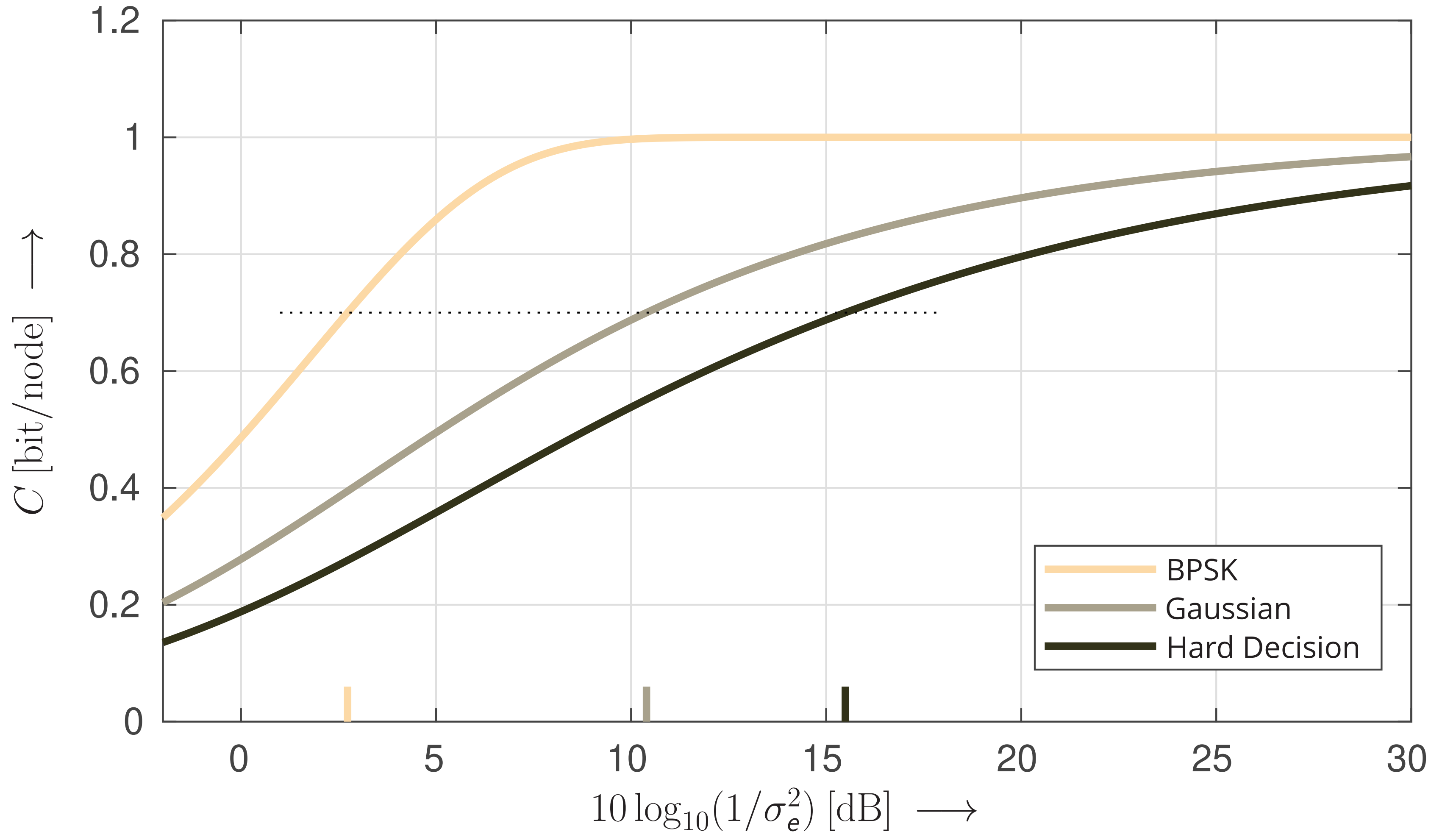
with $F \overset{\text{def}}{=} \frac{1}{\sqrt{1+\sigma_e^2}\,\sigma_e}$

$$Q(x) \overset{\text{def}}{=} \int_x^\infty \frac{1}{\sqrt{2\pi}}\, e^{-\frac{z^2}{2}}\, dz$$ (complementary Gaussian integral function)

Numerical Examples

Capacities over the Signal-to-Noise Ratio (in dB):

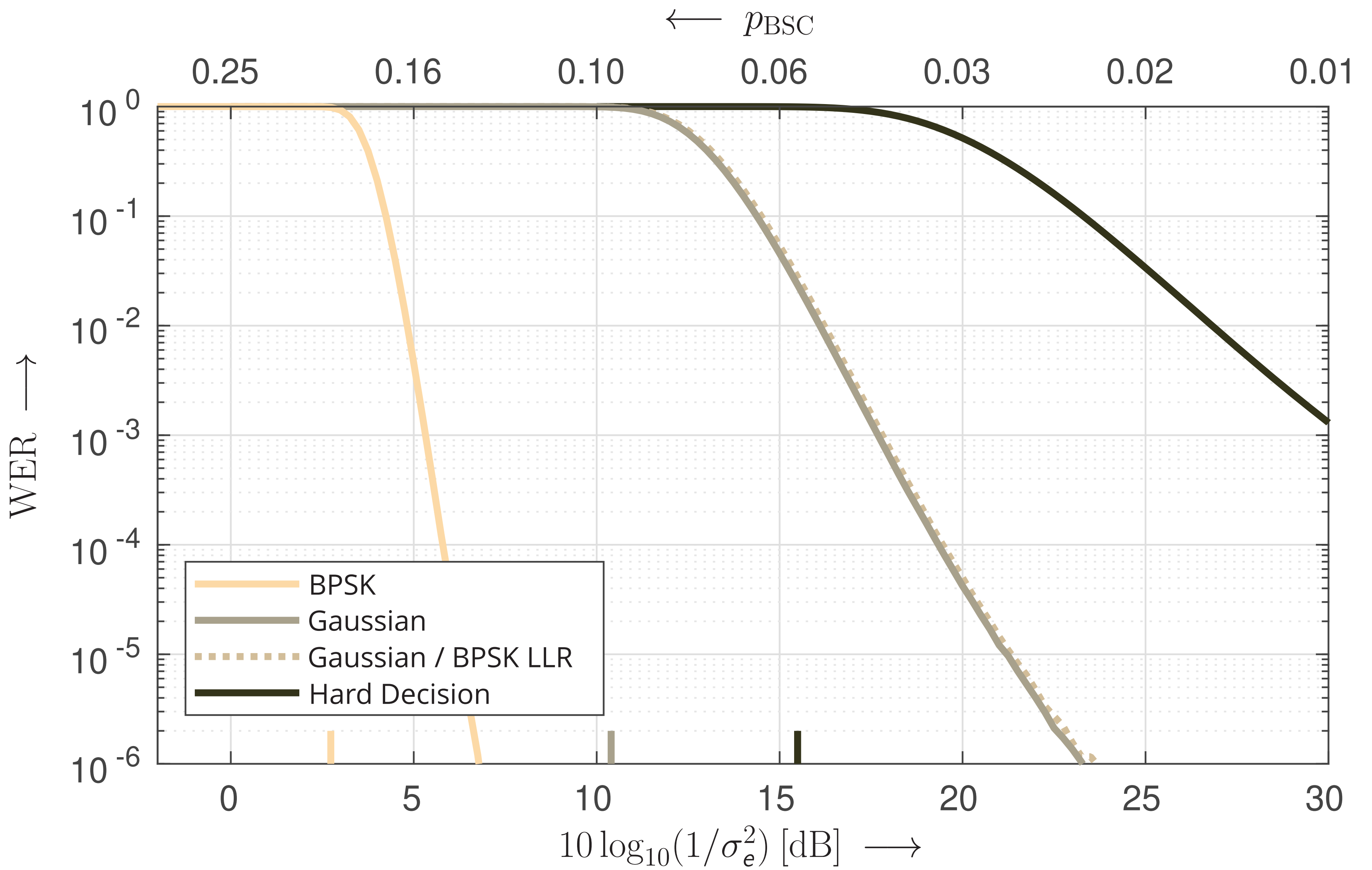
- BPSK
- Gaussian readout



Numerical Examples (II)

Word Error Ratio (WER) over the Signal-to-Noise Ratio (in dB):

- PUF nodes: 1024
  mess. length: 717
  rate: $R = 0.7$ $\left[\frac{\text{bit}}{\text{node}}\right]$
- Polar code
  - codelength $n = 1024$
  - rate $R = 0.7$

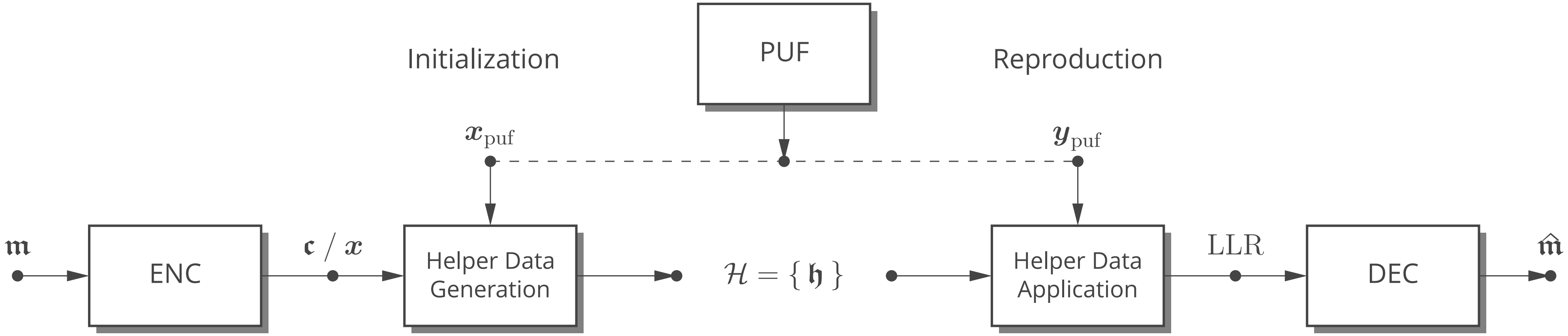


Coded Modulation and Shaping

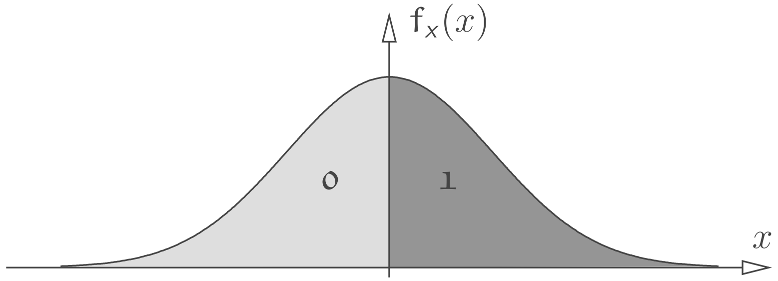
Situation

Binary Soft-Output PUF:

- generation of and communication via ***helper data***



- mapping bits to ***regions***



Challenge:

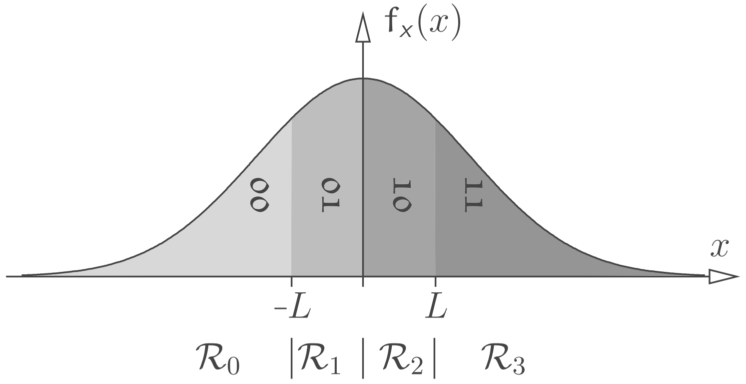
- increase code rate / size of the message $\mathfrak{m}$ — extract more than one bit per PUF node

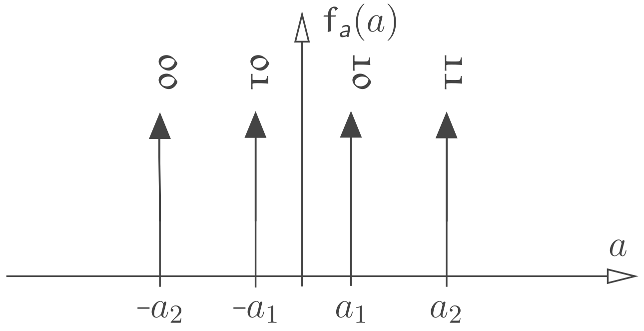
  ⇒ ***employ higher-order modulation / coded modulation***

Regions and Schemes

PUF Readout and Regions:

- regions for 4-ary signaling





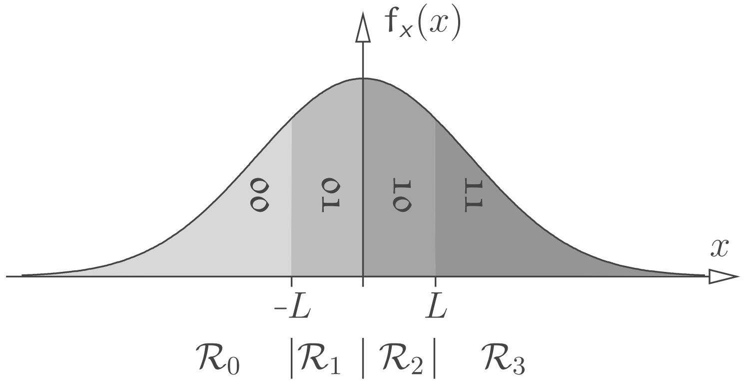
- regions $\mathcal{R}_\rho$
- natural labeling:
  - label $\mathfrak{c} = [\mathfrak{c}_1 \mathfrak{c}_0]$
  - region number $\rho = [\mathfrak{c}_1 \mathfrak{c}_0]_2$
- for $L = 0.675$ the regions are drawn with the same probability

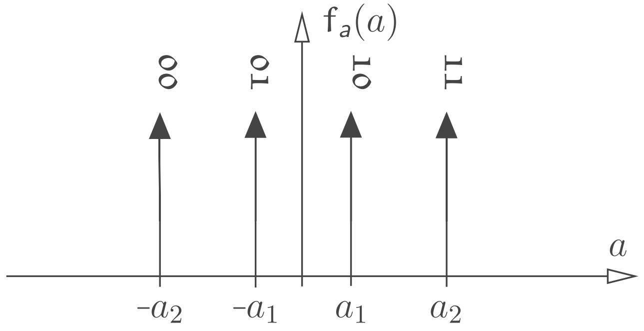
⇒ ***4-ary uniform scheme***

Regions and Schemes

PUF Readout and Regions:

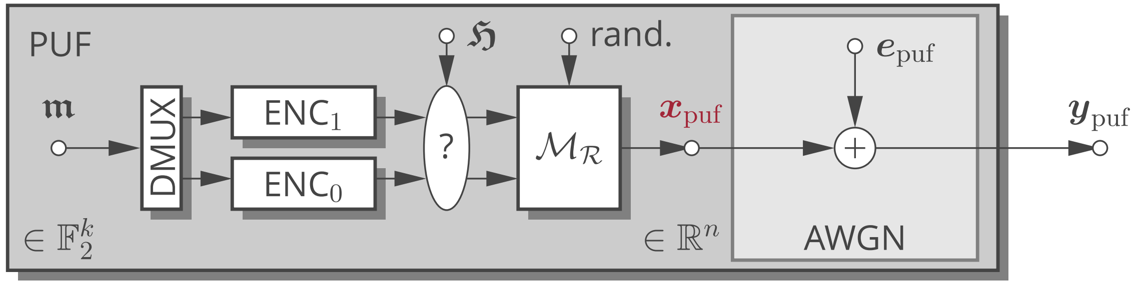
- regions for 4-ary signaling





Model of the PUF:

- we *imagine* a digital communication scheme

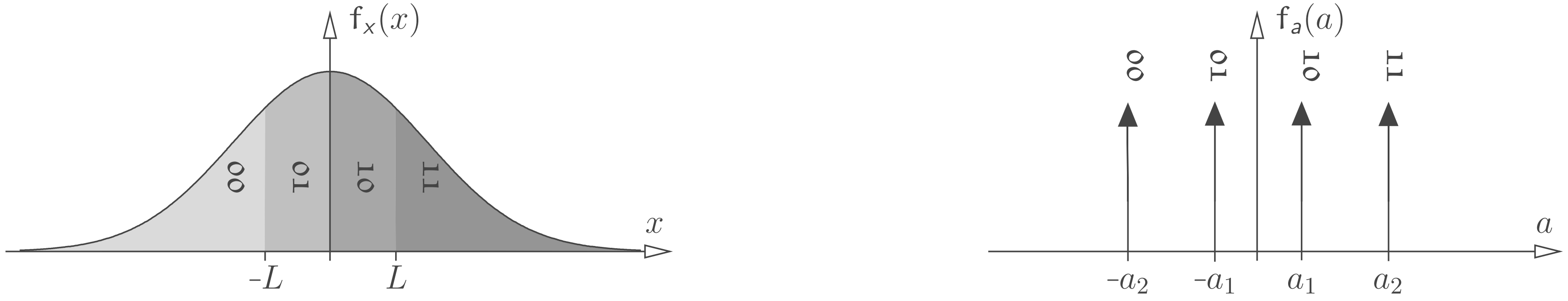


- ***mapping bits to regions*** — the actual number is drawn randomly according to a Gaussian pdf
- suited helper data scheme required

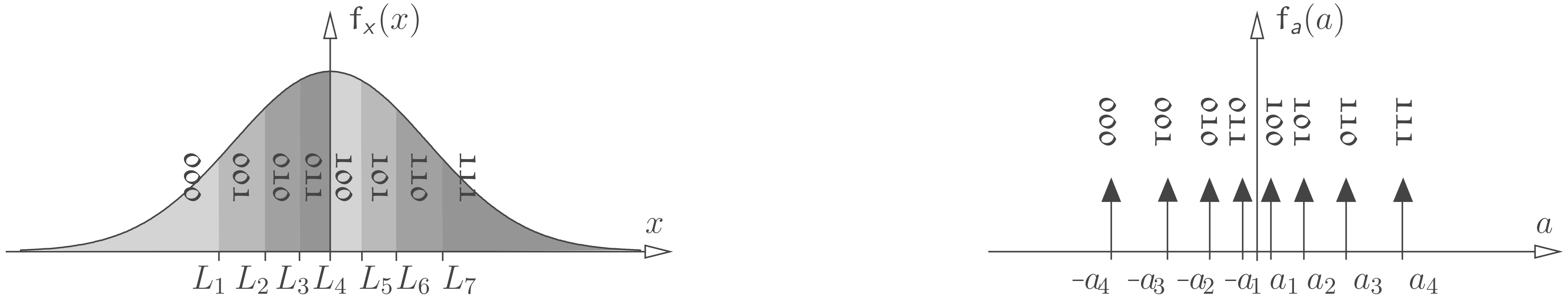
Regions and Schemes (II)

PUF Readout and Regions:

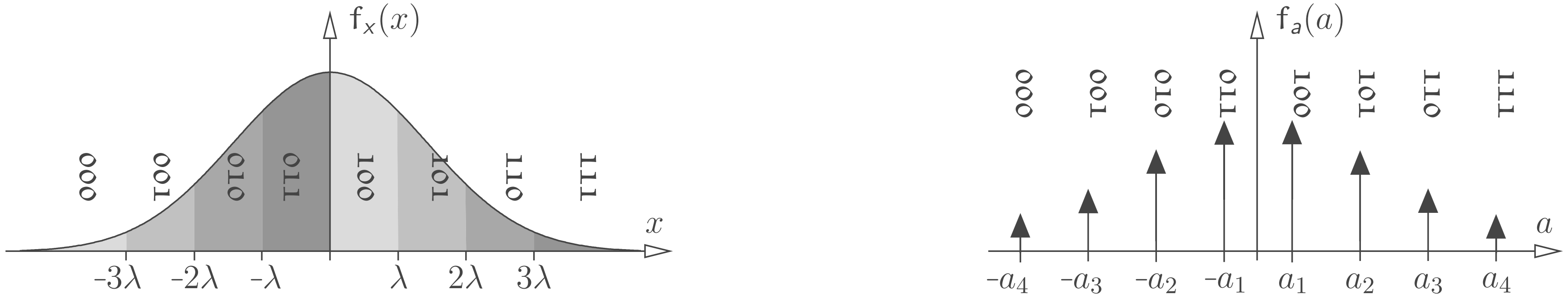
- 4-ary uniform signaling $(L = 0.675)$



- 8-ary uniform signaling



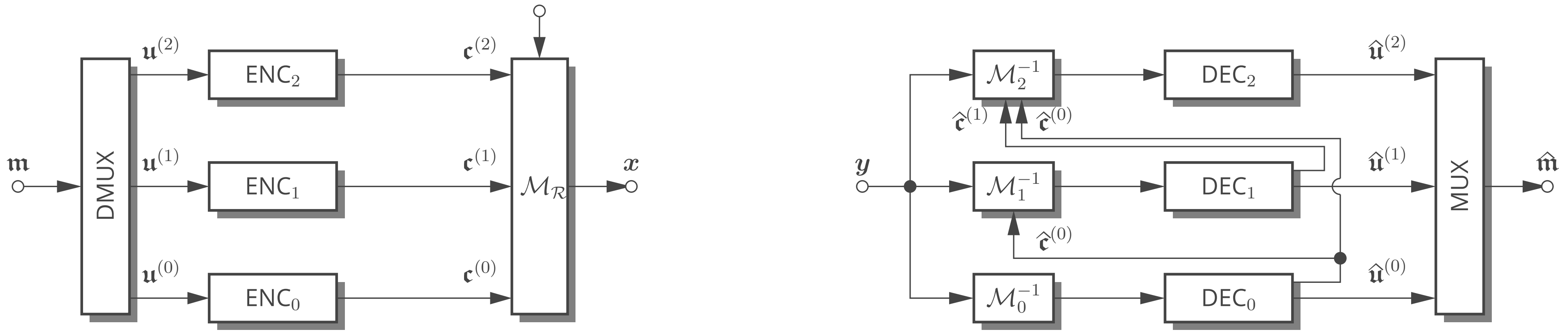
- 8-ary non-uniform signaling / shaping



Regions and Schemes (III)

Multilevel Encoder and Multistage Decoding: here: $M = 8$, $\mu = \log_2(M)$

- scheme for uniform signaling



Regions and Schemes (III)

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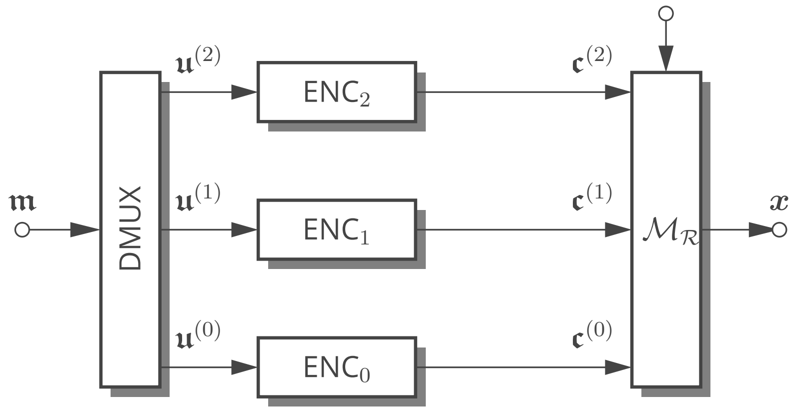
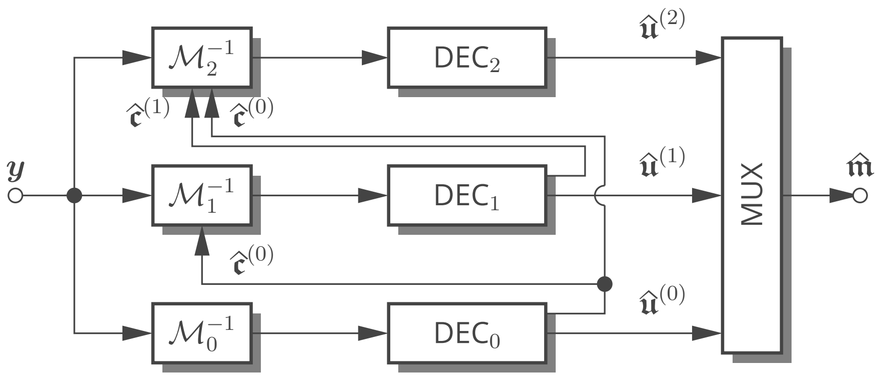




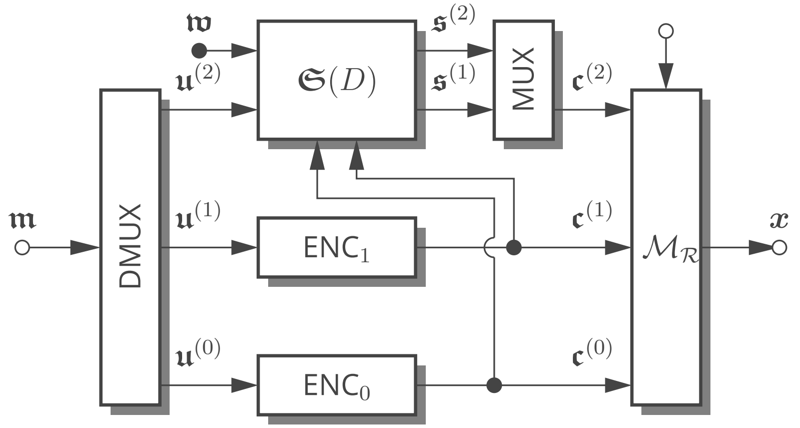
- specification by ***codematrix*** (code of length n)

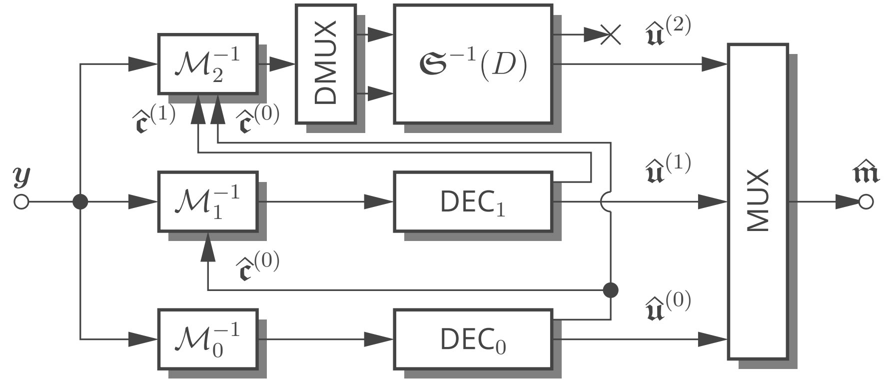
$$\mathfrak{m} \xrightarrow{\text{ENC}} \mathfrak{C} = \begin{array}{|c|c|c|c|c|c|c|}\hline \mathfrak{c}_{\mu\text{-}1,1} & \mathfrak{c}_{\mu\text{-}1,2} & \mathfrak{c}_{\mu\text{-}1,3} & \cdots\cdot & \mathfrak{c}_{\mu\text{-}1,i} & \cdots\cdot & \mathfrak{c}_{\mu\text{-}1,n} \\ \hline \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \hline \mathfrak{c}_{0,1} & \mathfrak{c}_{0,2} & \mathfrak{c}_{0,3} & \cdots\cdot & \mathfrak{c}_{0,i} & \cdots\cdot & \mathfrak{c}_{0,n} \\ \hline \end{array} = \begin{bmatrix} \mathfrak{c}^{(\mu-1)} \\ \vdots \\ \mathfrak{c}^{(0)} \end{bmatrix}$$

Region Number $\rho_1 \quad \rho_2 \quad \rho_3 \quad \cdots \quad \rho_i = [\mathfrak{c}_{\mu\text{-}1,i} \cdots \mathfrak{c}_{0,i}]_2 \quad \cdots \quad \rho_n$

Multilevel Encoder and Multistage Decoding: here: $M = 8$, $\mu = \log_2(M)$

- scheme with trellis shaping (highest level has rate $1/2$)





- specification by ***codematrix*** (code of length n)

$\mathfrak{m} \xrightarrow{\text{ENC}} \mathfrak{C} =$

$\mathfrak{c}_{\mu\text{-}1,1}$	$\mathfrak{c}_{\mu\text{-}1,2}$	$\mathfrak{c}_{\mu\text{-}1,3}$	$\cdots$	$\mathfrak{c}_{\mu\text{-}1,i}$	$\cdots$	$\mathfrak{c}_{\mu\text{-}1,n}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\mathfrak{c}_{0,1}$	$\mathfrak{c}_{0,2}$	$\mathfrak{c}_{0,3}$	$\cdots$	$\mathfrak{c}_{0,i}$	$\cdots$	$\mathfrak{c}_{0,n}$

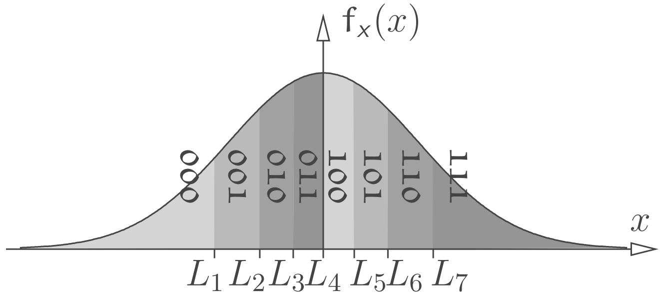
$= \begin{bmatrix} \mathfrak{c}^{(\mu-1)} \\ \vdots \\ \mathfrak{c}^{(0)} \end{bmatrix}$

Region Number $\rho_1 \quad \rho_2 \quad \rho_3 \quad \cdots \quad \rho_i = [\mathfrak{c}_{\mu\text{-}1,i} \cdots \mathfrak{c}_{0,i}]_2 \quad \cdots \quad \rho_n$

Numerical Examples

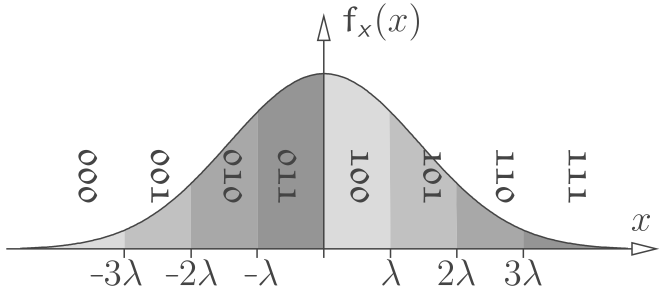
Capacities over the Signal-to-Noise Ratio (in dB):

- uniform:

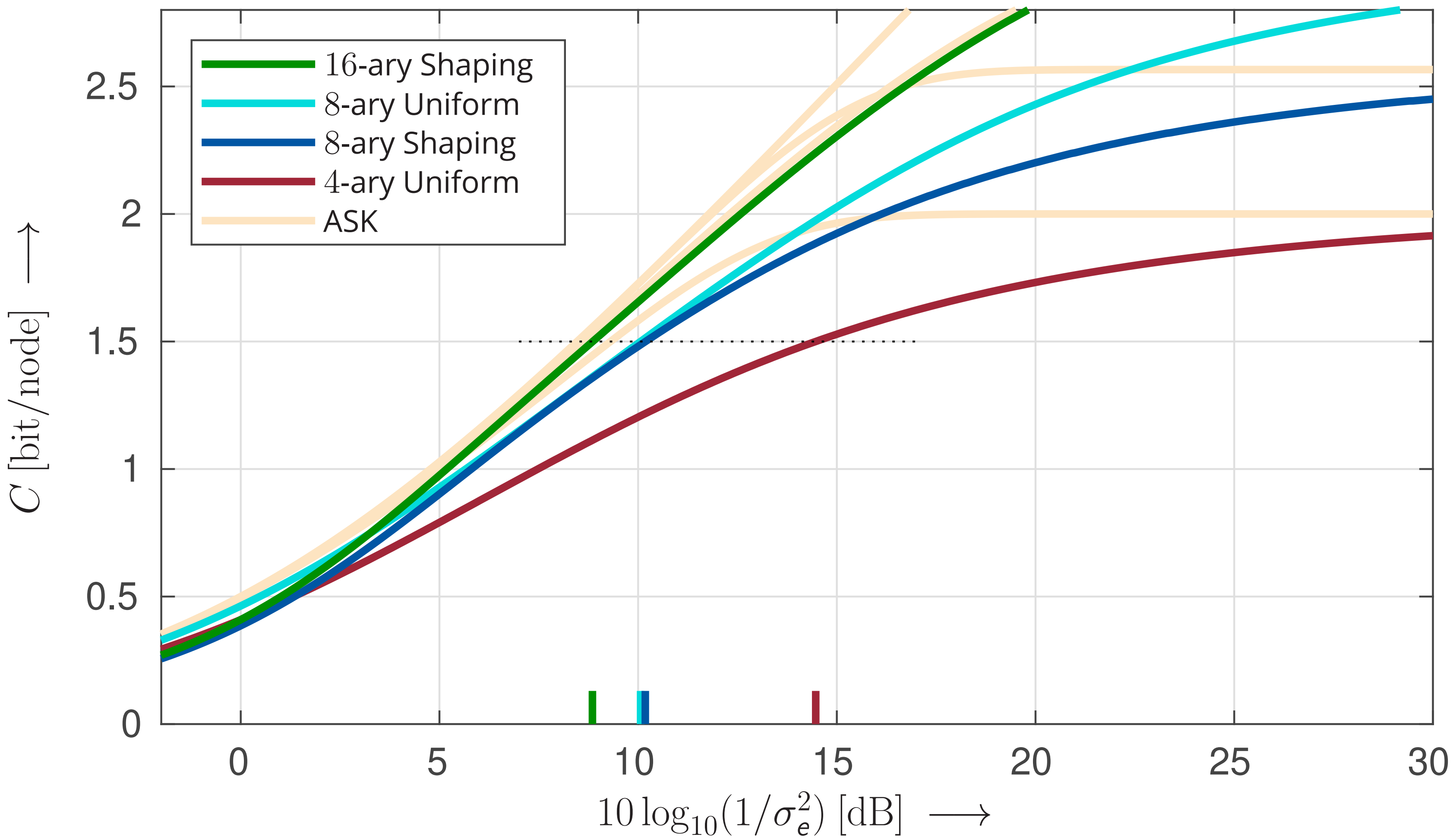


  - 4-ary Uniform
  - 8-ary Uniform

- shaping:

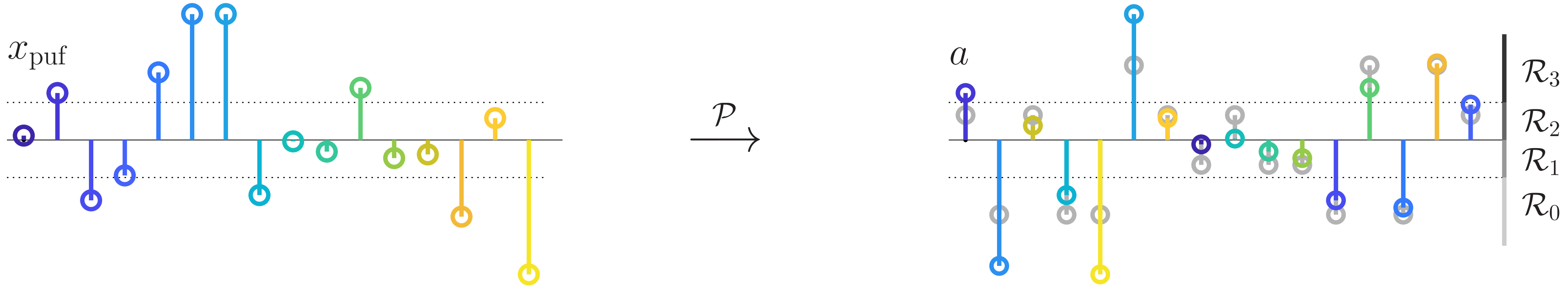


  - 8-ary Shaping: $\lambda = 0.70$
  - 16-ary Shaping: $\lambda = 0.35$
  - highest level: $R_{\mu\text{-}1} = 0.5$, hard decision



First Approach: generate a valid codeword in signal space

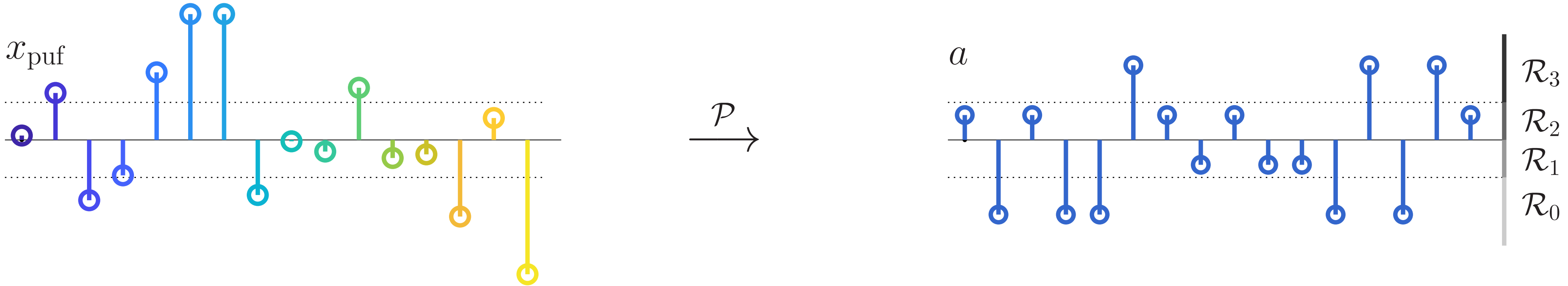
- employ ***permutation*** and ***sign flip*** [FM'22]
  - easy to implement
  - large number of bits required to store the helper data: $\approx n(1 + \log_2(n))$
  - no perfect match possible



Helper Data Scheme

First Approach: generate a valid codeword in signal space

- employ ***permutation*** and ***sign flip*** [FM'22]
  - easy to implement
  - large number of bits required to store the helper data: $\approx n(1+\log_2(n))$
  - no perfect match possible



$$\mathfrak{Q} = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \xleftarrow{\mathcal{H}} \mathfrak{C} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

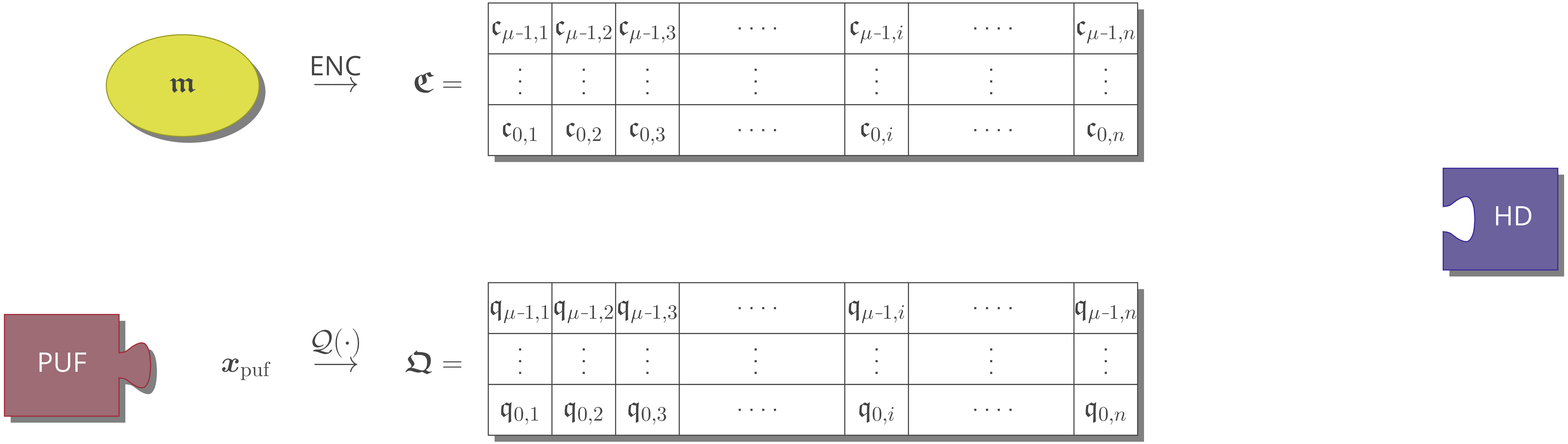
Better Approach: adapt LLR calculation

- employ a ***conversion*** of the region labels
  - applied element-wise
  - small number of bits required to store the helper data: $n\ \log_2(M)$
  - ideal LLR calculation

Helper Data Scheme (II)

Calculation of Helper Data: uniform signaling

- visualization

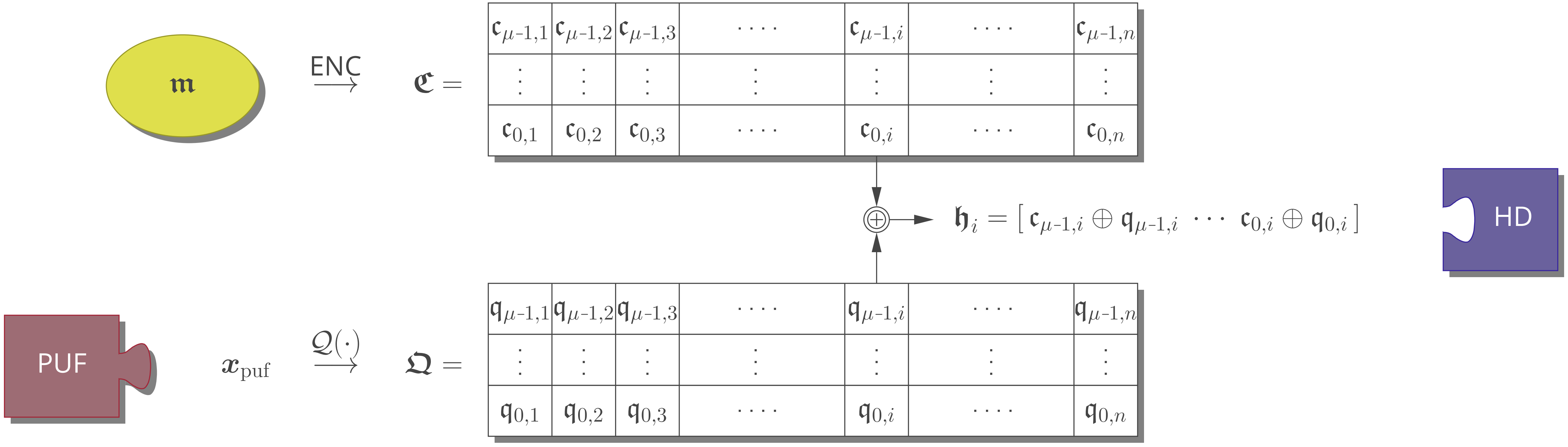


- $[\mathfrak{c}_{\mu\text{-}1,i} \ \cdots \ \mathfrak{c}_{0,i}]_2$: desired codesymbols
  $[\mathfrak{q}_{\mu\text{-}1,i} \ \cdots \ \mathfrak{q}_{0,i}]_2$: obtained by quantization $\mathcal{Q}(\cdot)$

Helper Data Scheme (II)

Calculation of Helper Data: uniform signaling

- visualization



$\mathfrak{c}_{\mu\text{-}1,1}$	$\mathfrak{c}_{\mu\text{-}1,2}$	$\mathfrak{c}_{\mu\text{-}1,3}$	$\cdots$	$\mathfrak{c}_{\mu\text{-}1,i}$	$\cdots$	$\mathfrak{c}_{\mu\text{-}1,n}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\mathfrak{c}_{0,1}$	$\mathfrak{c}_{0,2}$	$\mathfrak{c}_{0,3}$	$\cdots$	$\mathfrak{c}_{0,i}$	$\cdots$	$\mathfrak{c}_{0,n}$

$\mathfrak{q}_{\mu\text{-}1,1}$	$\mathfrak{q}_{\mu\text{-}1,2}$	$\mathfrak{q}_{\mu\text{-}1,3}$	$\cdots$	$\mathfrak{q}_{\mu\text{-}1,i}$	$\cdots$	$\mathfrak{q}_{\mu\text{-}1,n}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\mathfrak{q}_{0,1}$	$\mathfrak{q}_{0,2}$	$\mathfrak{q}_{0,3}$	$\cdots$	$\mathfrak{q}_{0,i}$	$\cdots$	$\mathfrak{q}_{0,n}$

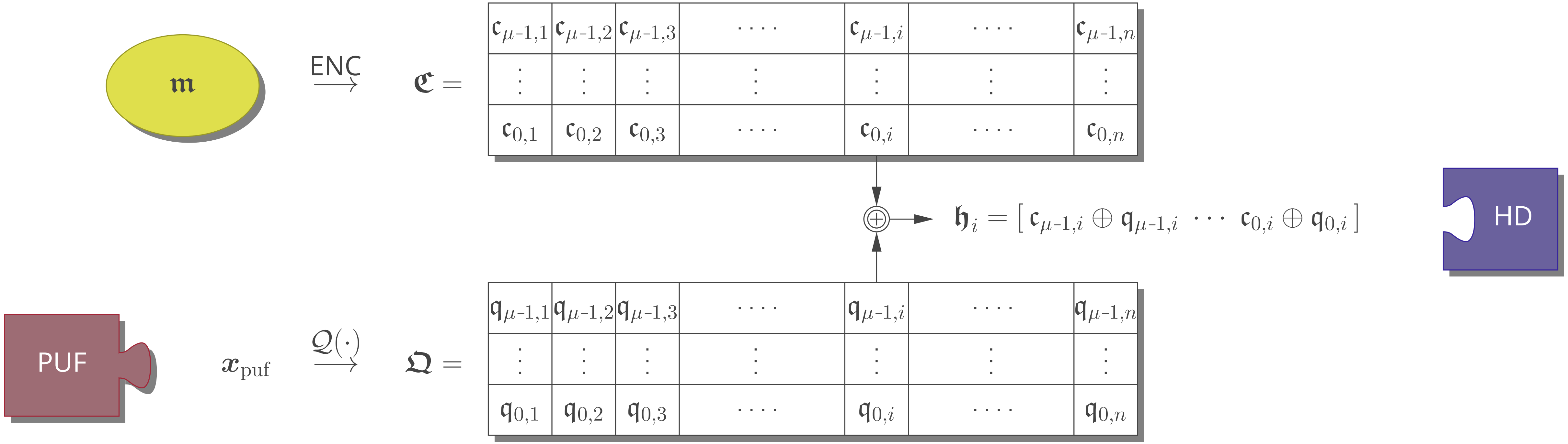
- $[\mathfrak{c}_{\mu\text{-}1,i} \;\cdots\; \mathfrak{c}_{0,i}]_2$: desired codesymbols
  $[\mathfrak{q}_{\mu\text{-}1,i} \;\cdots\; \mathfrak{q}_{0,i}]_2$: obtained by quantization $\mathcal{Q}(\cdot)$

- helper data: $\mathfrak{H} = \mathfrak{C} \oplus \mathfrak{Q}$

Helper Data Scheme (II)

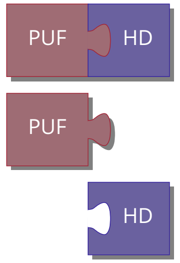
Calculation of Helper Data: uniform signaling

- visualization



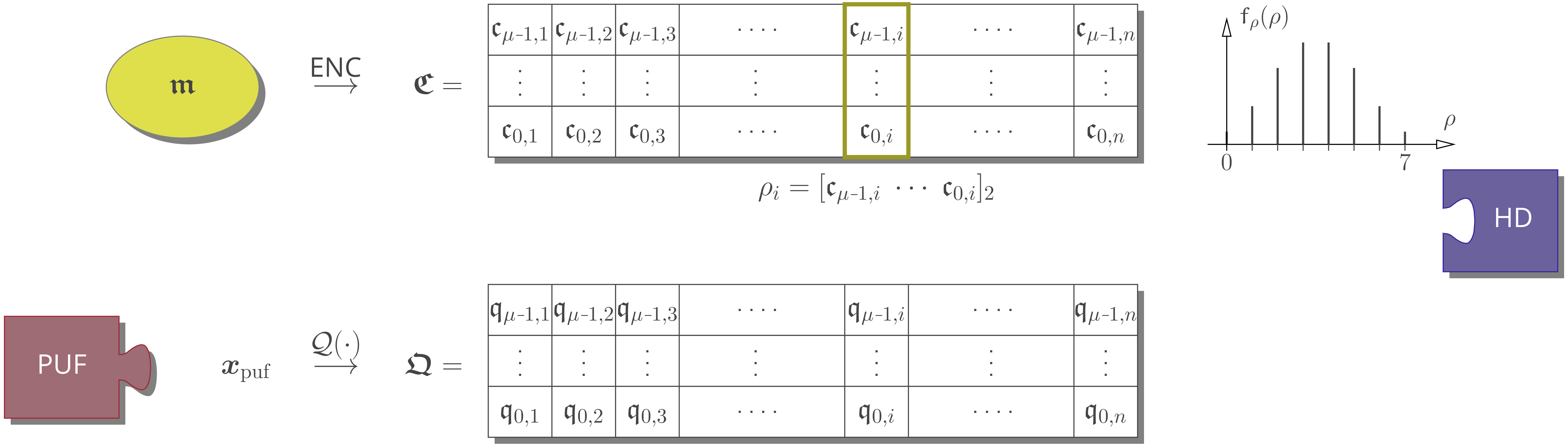
Security: it can be shown

- message can be decoded when knowing the PUF readout and the helper data
- no leakage when knowing the PUF readout only
- no leakage when knowing the helper data only



Calculation of Helper Data: shaped signaling

- visualization

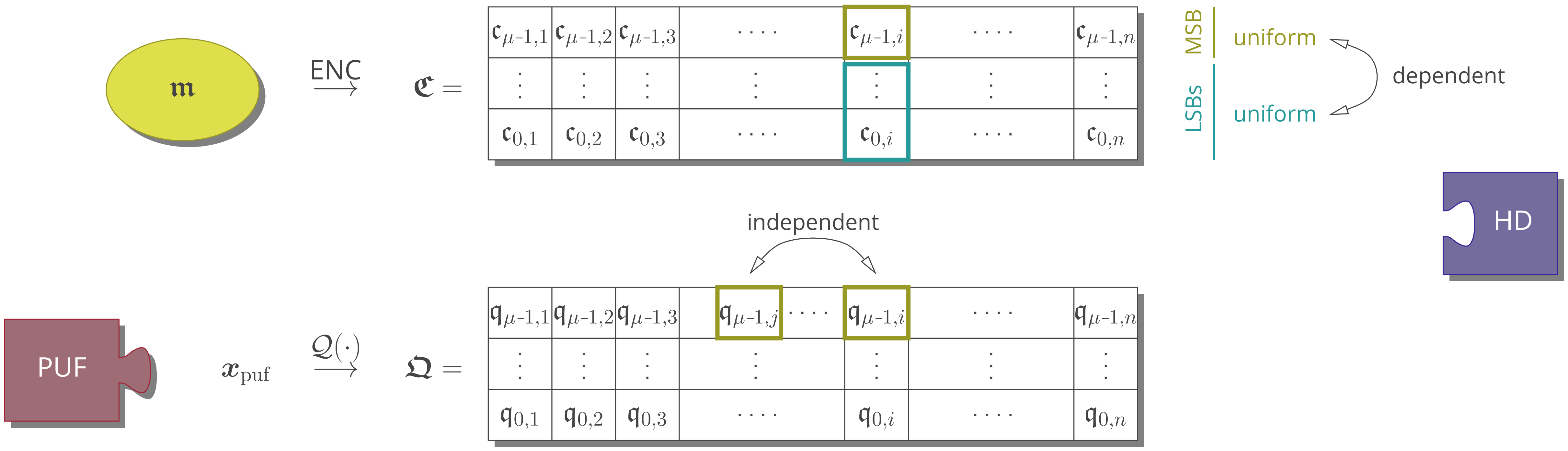


Problem:

- region numbers not uniformly distributed — leakage

Calculation of Helper Data: shaped signaling

- visualization



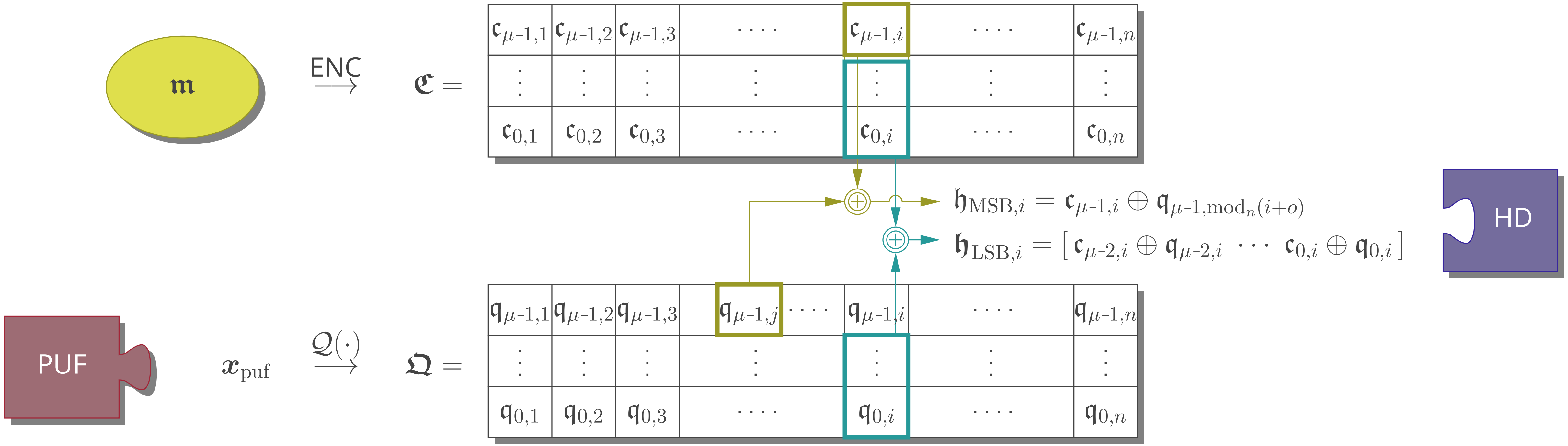
Problem:

- region numbers not uniformly distributed — leakage

Helper Data Scheme (III)

Calculation of Helper Data: shaped signaling

- visualization



Problem:

- region numbers not uniformly distributed — leakage

Solution:

- $\mathfrak{c}_{\mu-1,i} \oplus \mathfrak{q}_{\mu-1,i+o}$ independent on $[\mathfrak{c}_{\mu-2,i} \;\cdots\; \mathfrak{c}_{0,i}]$

Optimum Decoding

LLR Calculation: conversion helper scheme

- PUF readout $\boldsymbol{y}_{\text{puf}} = [y_{\text{puf},1}, \ldots, y_{\text{puf},n}]$
- LLR for label bit $\mathfrak{c}_{0,i}$

$$\text{LLR}(\mathfrak{c}_{0,i}) \;=\; \log\left(\frac{\sum_{\forall \mathfrak{q},\, \mathfrak{q}_{0,i}=\mathfrak{0} \,\oplus\, \mathfrak{h}_{0,i}} \Delta\text{Q}(y_{\text{puf},i}, \mathcal{R}_{\mathfrak{q}})}{\sum_{\forall \mathfrak{q},\, \mathfrak{q}_{0,i}=\mathfrak{1} \,\oplus\, \mathfrak{h}_{0,i}} \Delta\text{Q}(y_{\text{puf},i}, \mathcal{R}_{\mathfrak{q}})}\right)$$

- definition

$$\Delta\text{Q}(y, \mathcal{R}_{\mathfrak{c}}) \overset{\text{def}}{=} \text{Q}(D\,L_\rho - F\,y) - \text{Q}(D\,L_{\rho+1} - F\,y)$$

with $\mathcal{R}_{\mathfrak{c}} = \mathcal{R}_{[\,\mathfrak{c}_{\mu\text{-}1}\,\cdots\,\mathfrak{c}_0\,]}$ — lower limit L_ρ; upper limit $L_{\rho+1}$, $\rho = [\,\mathfrak{c}_{\mu\text{-}1}\cdots\mathfrak{c}_0\,]_2$

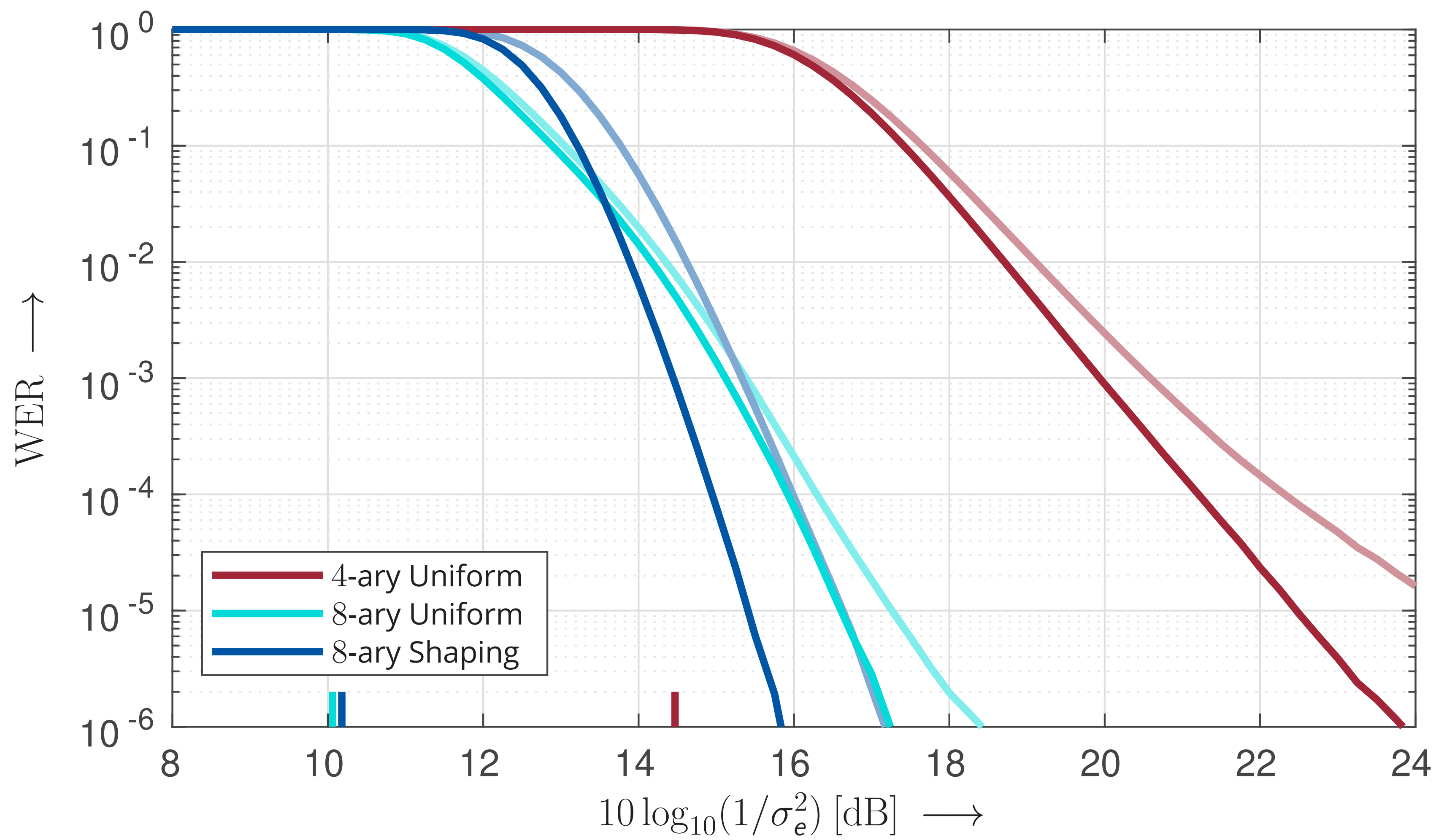
$$F \overset{\text{def}}{=} \frac{1}{\sqrt{1+\sigma_e^2}\,\sigma_e}, \qquad D \overset{\text{def}}{=} \frac{\sqrt{1+\sigma_e^2}}{\sigma_e}$$

$$\text{Q}(x) \overset{\text{def}}{=} \int_x^\infty \frac{1}{\sqrt{2\pi}}\,\text{e}^{-\frac{z^2}{2}}\,\text{d}z$$ (complementary Gaussian integral function)

Numerical Examples

Word Error Ratio (WER) over the Signal-to-Noise Ratio (in dB):

- PUF nodes: 1024
  mess. length: 1536
  rate: $R = 1.5 \; [\frac{\text{bit}}{\text{node}}]$
- Polar code
  - codelength $n = 1024$
- MLC
  - 4-ary Uniform:
    $k_0 = 523,\; k_1 = 1013$
  - 8-ary Uniform:
    $k_0 = 106,\; k_1 = 439,\; k_2 = 991$
  - 8-ary Shaping:
    $k_0 = 100,\; k_1 = 924,\; k_2 = 512$
- permutation vs. conversion

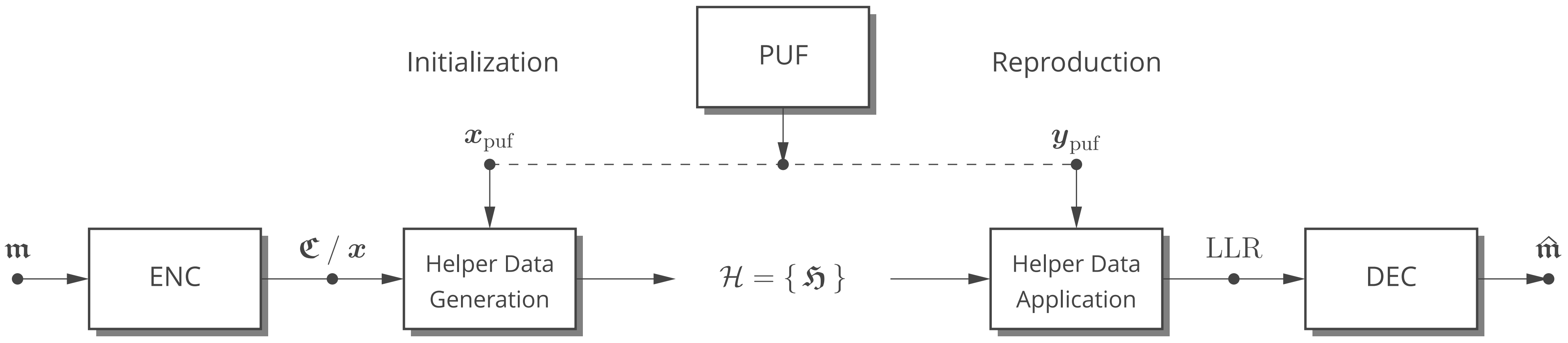


Helper Data for Improved Decoding

Situation

Coded Modulation / Shaping for PUFs:

- generation of and communication via ***helper data***



- helper data enables decoding in the first place

Improvement:

- recently, a *two-metric helper data scheme* was proposed [DGS'19], [TKDP'21]
  - two possible quantizers are available at reconstruction (uncoded case)
  - reference PUF readout determines which quantizer should be used (per PUF node)
  - these binary flags establish the helper data

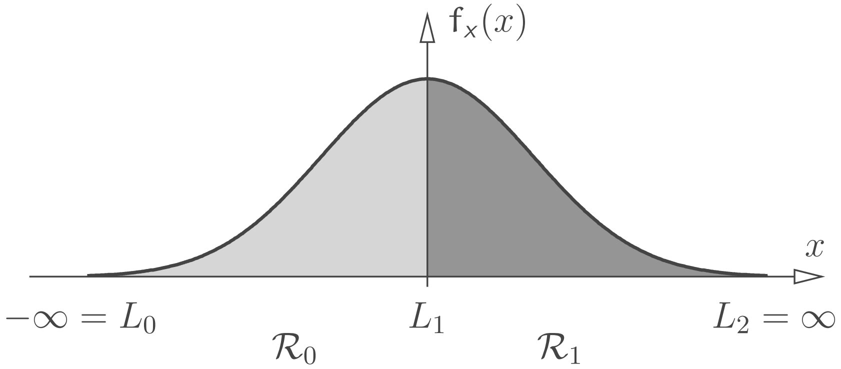
⇒ ***generalization to M-ary coded modulation***

Regions

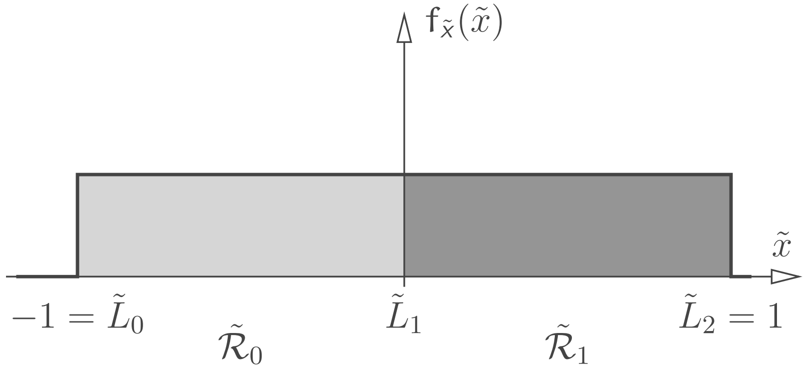
$\mathrm{erf}(z) \overset{\text{def}}{=} \frac{2}{\sqrt{\pi}} \int_0^z \mathrm{e}^{-t^2}\,\mathrm{d}t$

Regions for Uniform Signaling:

- visualization — $M = 2,\ S = 1$



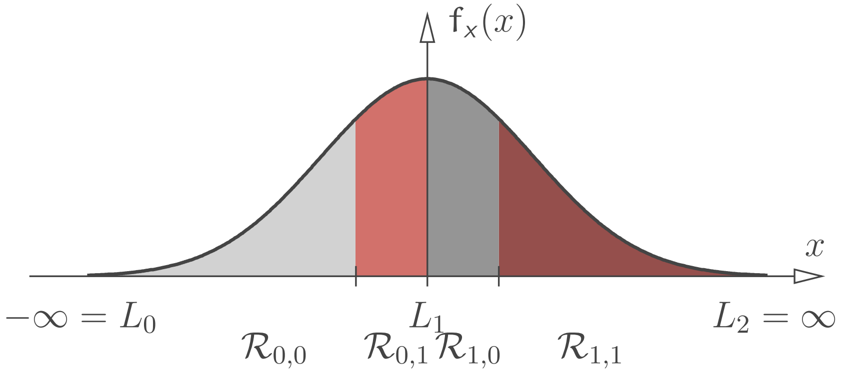
$$\tilde{x} = \mathrm{erf}(x/\sqrt{2})$$
$$x = \sqrt{2}\,\mathrm{erf}^{-1}(\tilde{x})$$

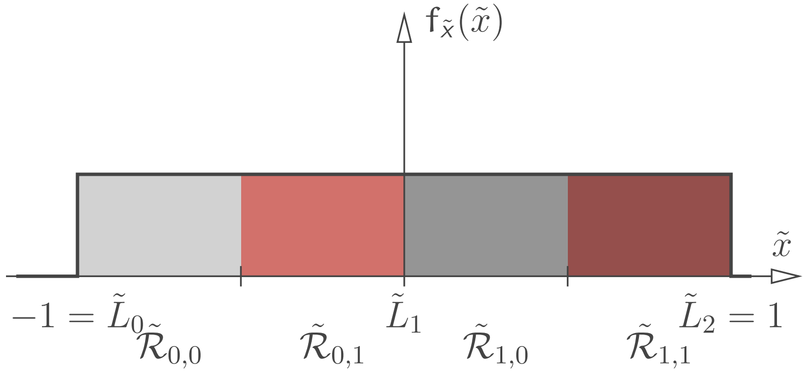


Regions

Regions for Uniform Signaling:

- visualization — $M = 2,\ S = 2$

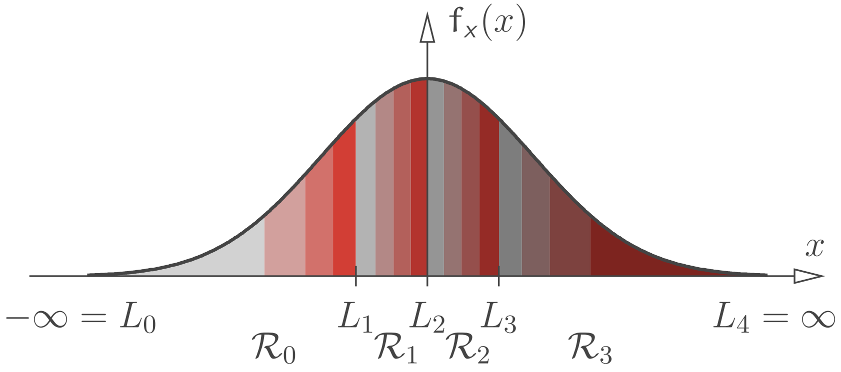


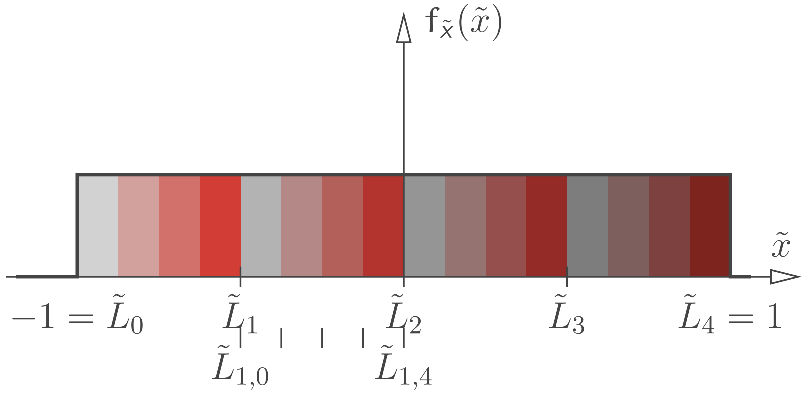


Regions

Regions for Uniform Signaling:

- visualization — $M = 4,\ S = 4$





- region limits for M-ary S-metric scheme

$$\tilde{L}_{\rho,s} = \tilde{L}_\rho + \frac{\tilde{L}_{\rho+1} - \tilde{L}_\rho}{S}\, s\,, \qquad \begin{array}{c} \rho = 0, \ldots, M-1 \\ s = 0, \ldots, S \end{array}$$

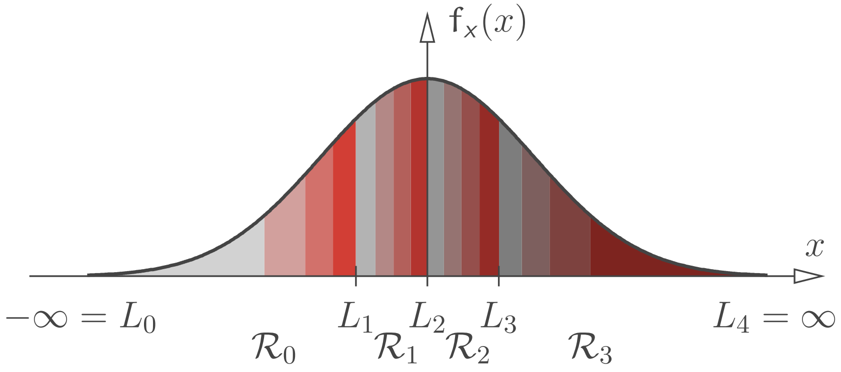
Initialization Phase:

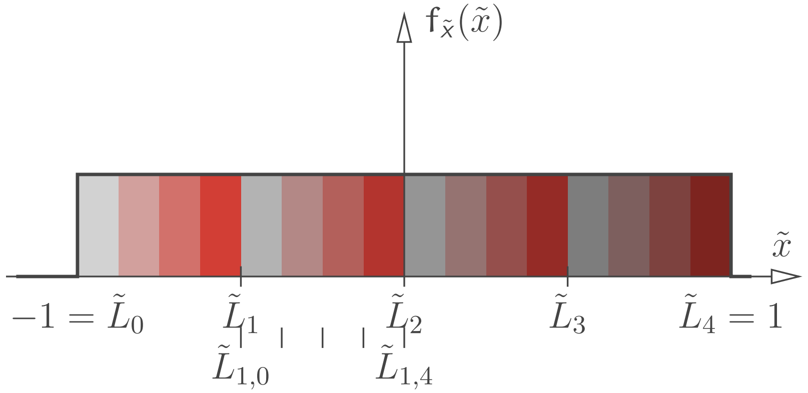
- quantization of the reference PUF readout $\boldsymbol{x}_{\text{puf}}$ (limits $L_{\rho,s}$) $\Rightarrow$ ***region ρ and subregion s***
- total helper data $\mathcal{H} = \{\, \boldsymbol{\mathfrak{H}},\ \boldsymbol{s} \,\}$ $\Rightarrow$ $n\left(\log_2(M) + \log_2(S)\right)$ bits

Regions

Regions for Uniform Signaling:

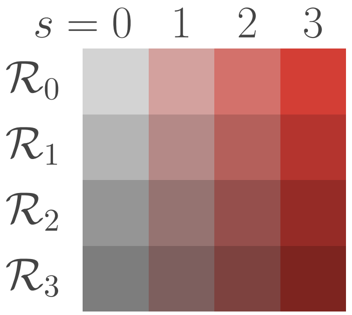
- visualization — $M = 4, \; S = 4$





Security:

- due to construction



$$\Pr\{s\} = \frac{1}{S}$$

and

$$p_{\rho,s} = \Pr\{x \in \mathcal{R}_{\rho,s}\} = \Pr\{x \in \mathcal{R}_\rho\} \, \frac{1}{S}$$

⇒ subregion number s is uniformly distributed

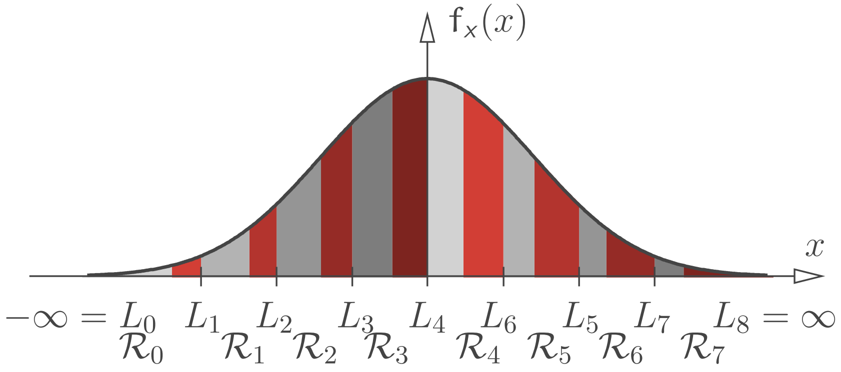
⇒ region number ρ and subregion number s are independent

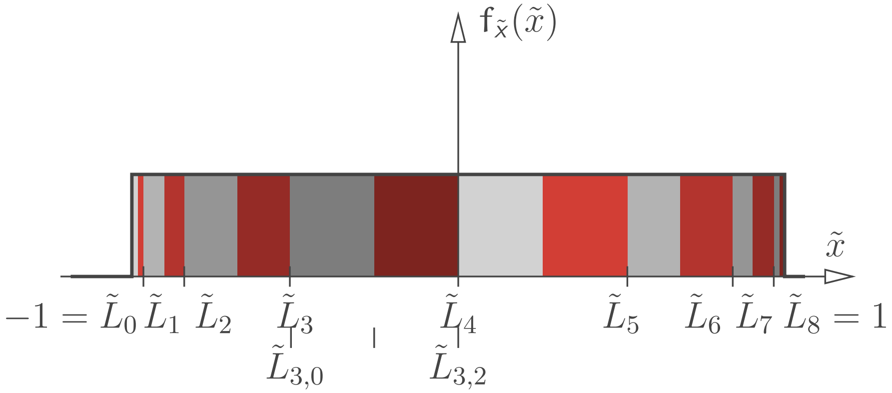
⇒ ***no leakage***

Regions

Regions for Shaping:

- visualization — $M = 8,\ S = 2$





Security:

- due to construction

$$\Pr\{s\} = \frac{1}{S}$$

and

$$p_{\rho,s} = \Pr\{x \in \mathcal{R}_{\rho,s}\} = \Pr\{x \in \mathcal{R}_{\rho}\}\,\frac{1}{S}$$

⇒ subregion number s is uniformly distributed

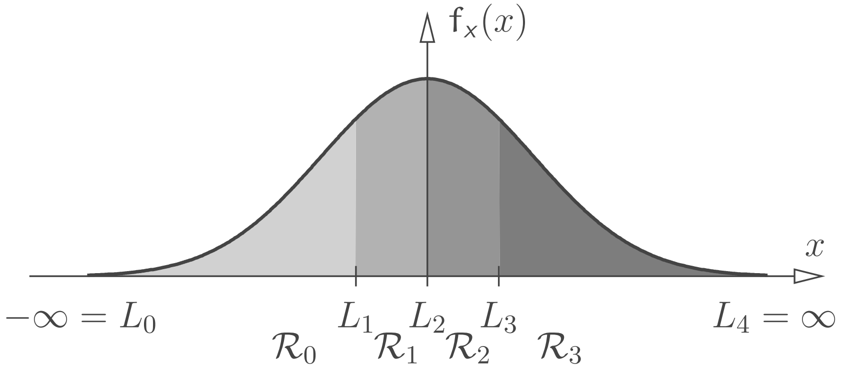
⇒ region number ρ and subregion number s are independent

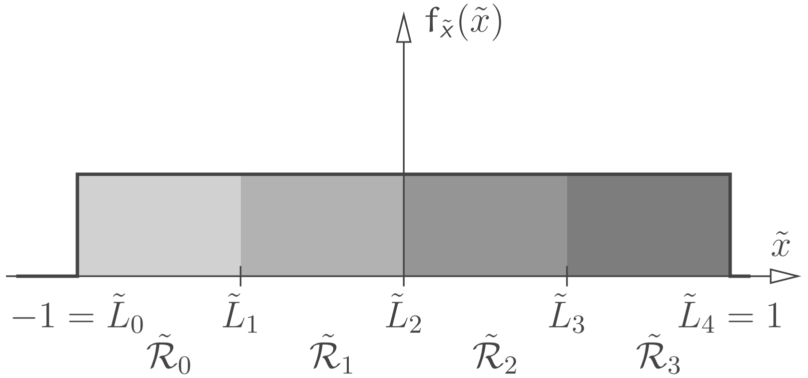
⇒ ***no leakage***

Constellations

Active Constellation: $M = 4$

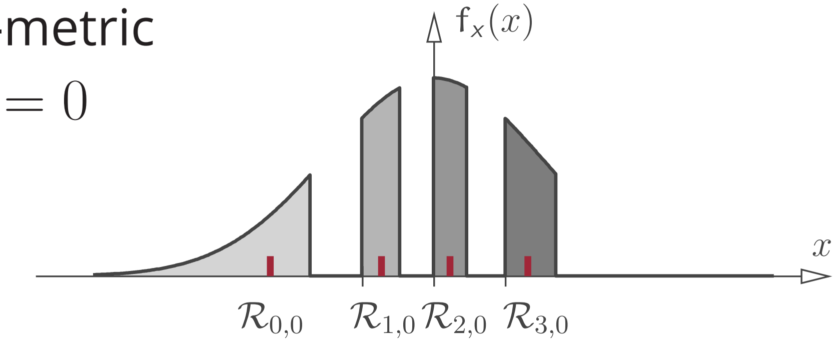
- conventional

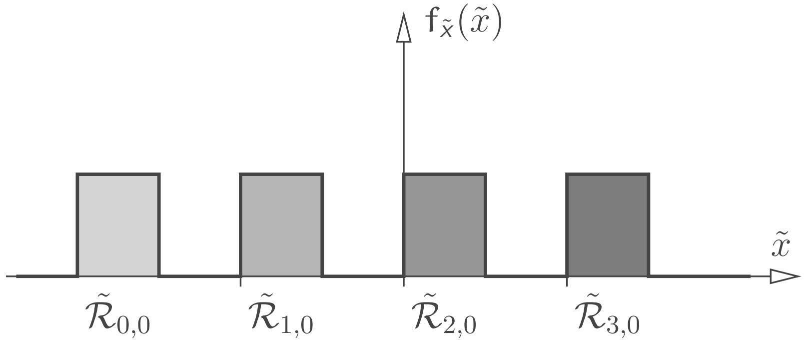




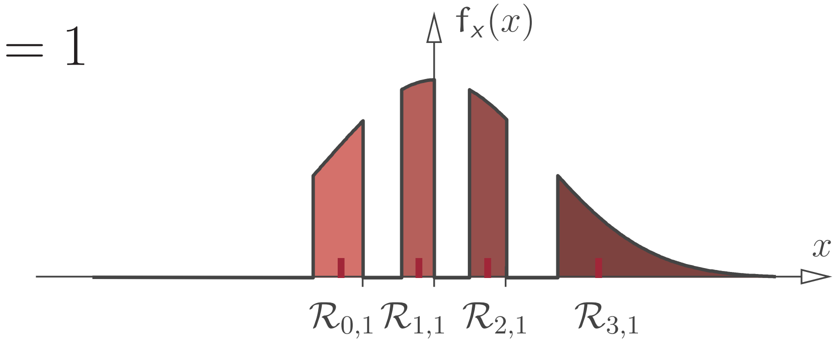
- 2-metric

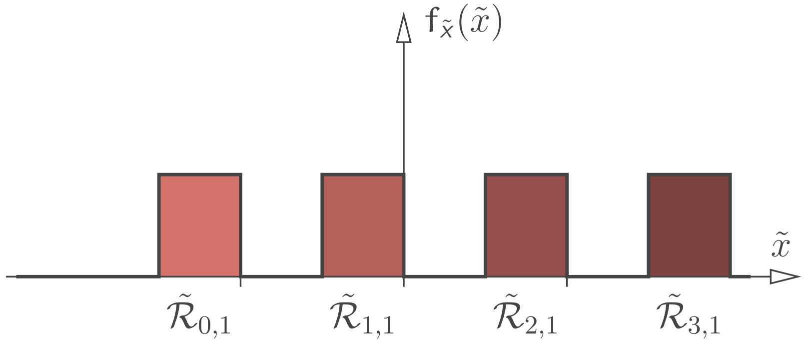
$s = 0$





$s = 1$

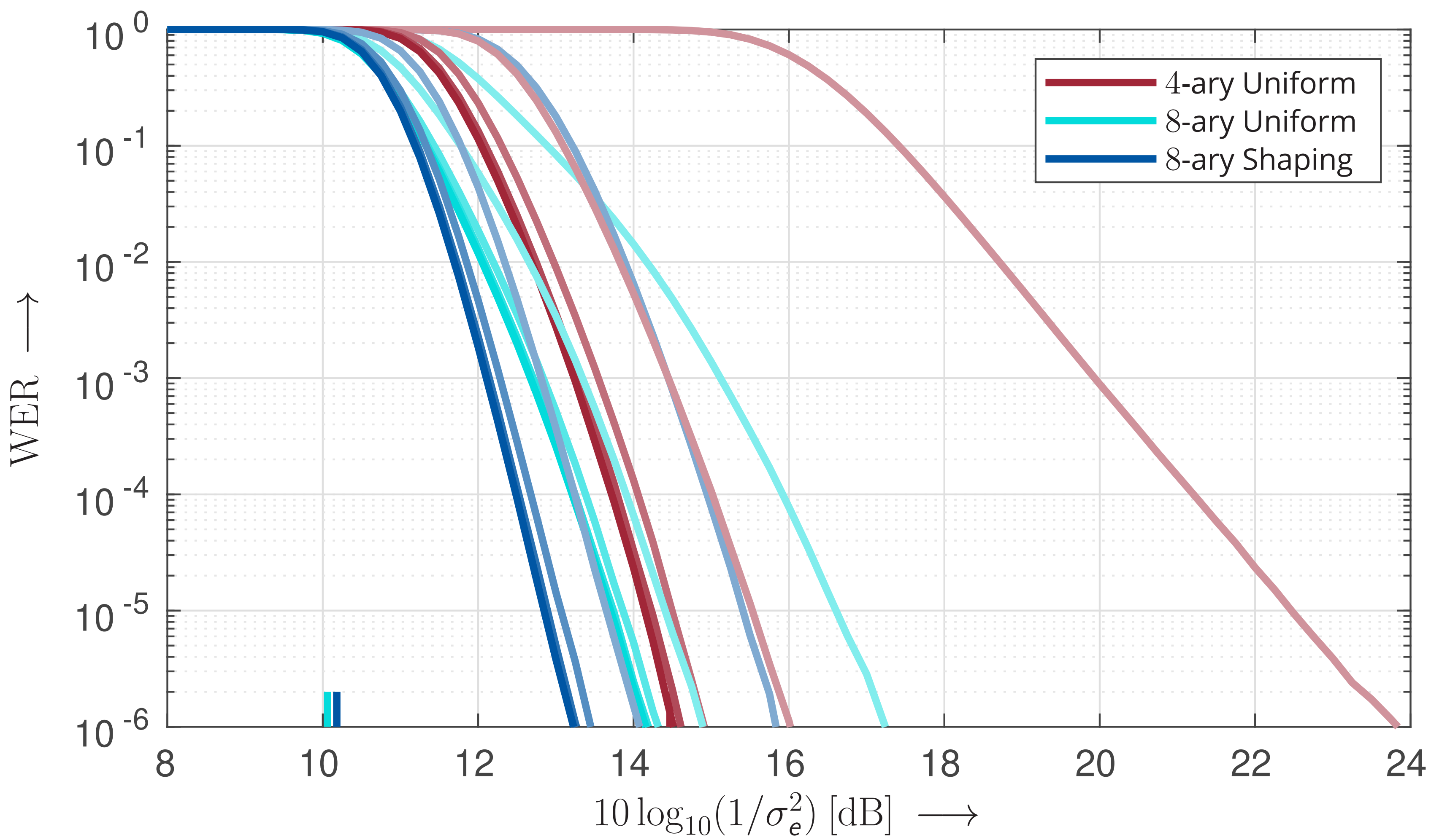




Numerical Examples

Word Error Ratio (WER) over the Signal-to-Noise Ratio (in dB):

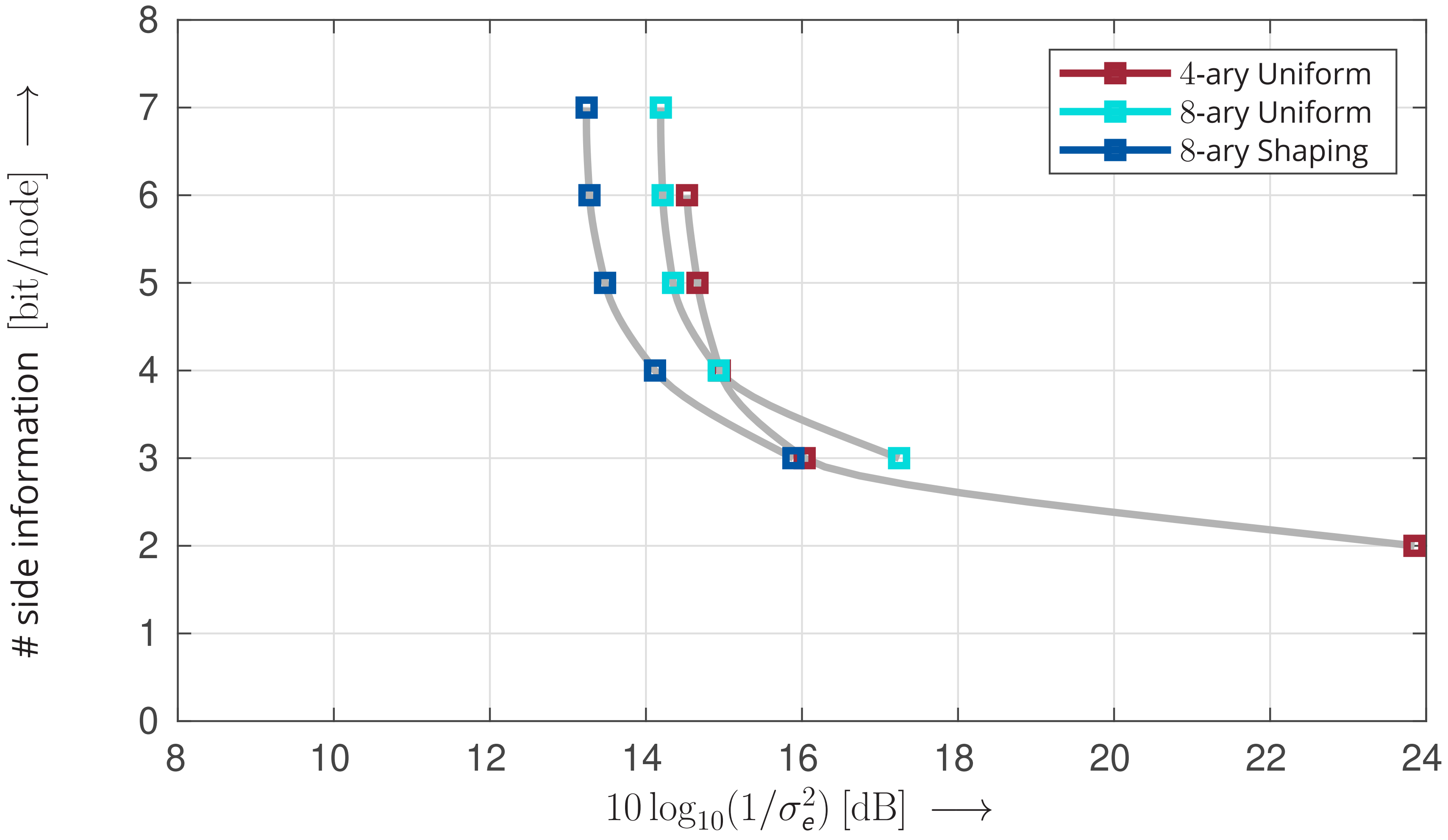
- PUF nodes: 1024
  mess. length: 1536
  rate: $R = 1.5 \ [\frac{\text{bit}}{\text{node}}]$
- Polar code
  - codelength $n = 1024$
- MLC
- conversion helper scheme
- $S = 1,\ 2,\ 4,\ 8,\ 16$



Numerical Examples (II)

 # Side Information $[\mathrm{bit/node}]$ over Required Signal-to-Noise Ratio (in dB):

- PUF nodes: 1024
  mess. length: 1536
  rate: $R = 1.5 \quad [\frac{\mathrm{bit}}{\mathrm{node}}]$
- $\mathrm{WER} = 10^{-6}$



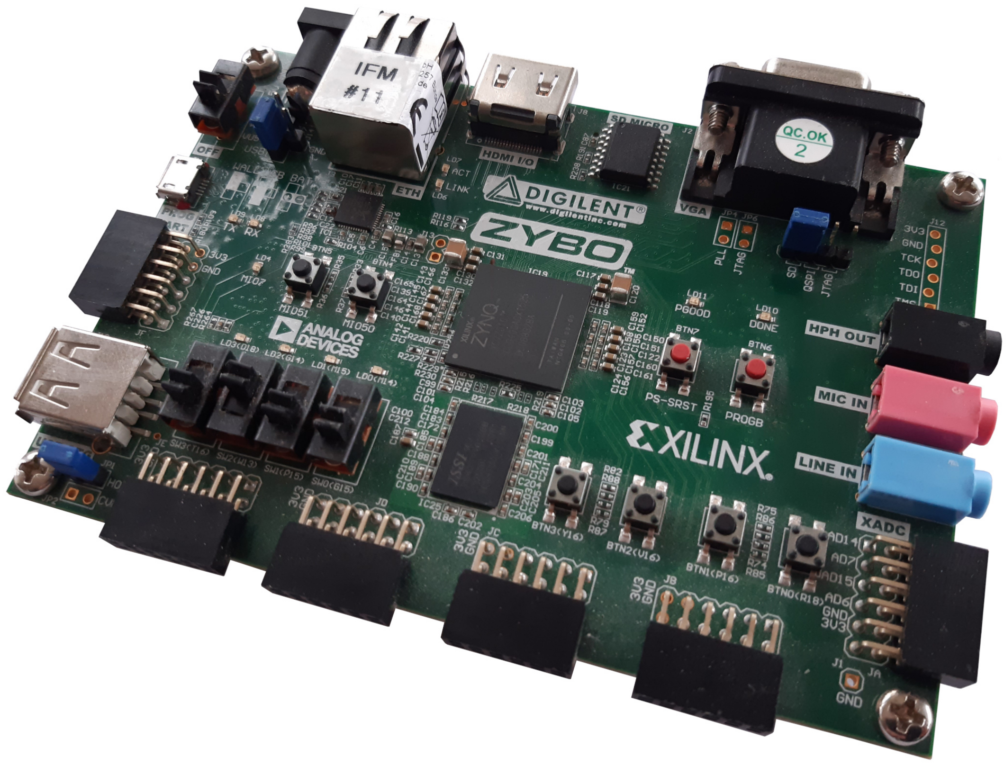
FPGA Implementation

FPGA Implementation

Specification:

[HMBO'19]

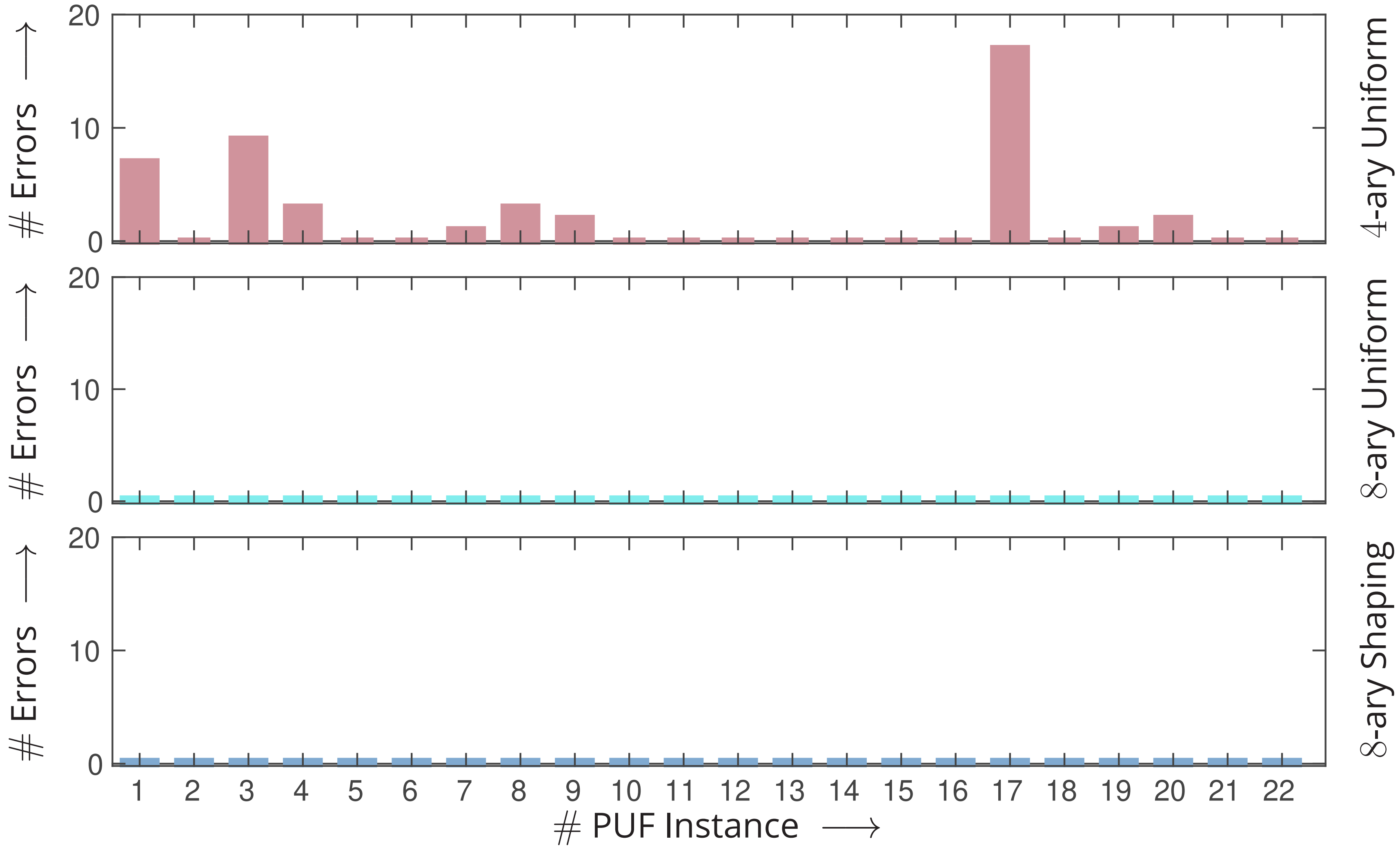
- ROPUFs implemented on XILINX FPGAs
  at the Institute of Microelectronics
- 22 instances (evaluation boards) available
- each comprising 3800 ROs
- $n = 1024$ disjoint pairs of ROs randomly selected
- temperature from $-10\ °\mathrm{C}$ to $50\ °\mathrm{C}$
  (in steps of $10\ °\mathrm{C}$)
- reference readout $\boldsymbol{x}_{\mathrm{ref}}$:
  average of 10 readouts at $20\ °\mathrm{C}$
- $10{,}000$ readouts per PUF instance and temperature
  (in total $70{,}000$ readouts per PUF instance)
- schemes
  - 4-ary uniform
  - 8-ary uniform
  - 8-ary shaping



FPGA Implementation (II)

Number of Word Errors: per 70,000 readouts

- PUF nodes: 1024
  mess. length: 1536
  rate: $R = 1.50$ $\left[\frac{\text{bit}}{\text{node}}\right]$
- $S = 1$



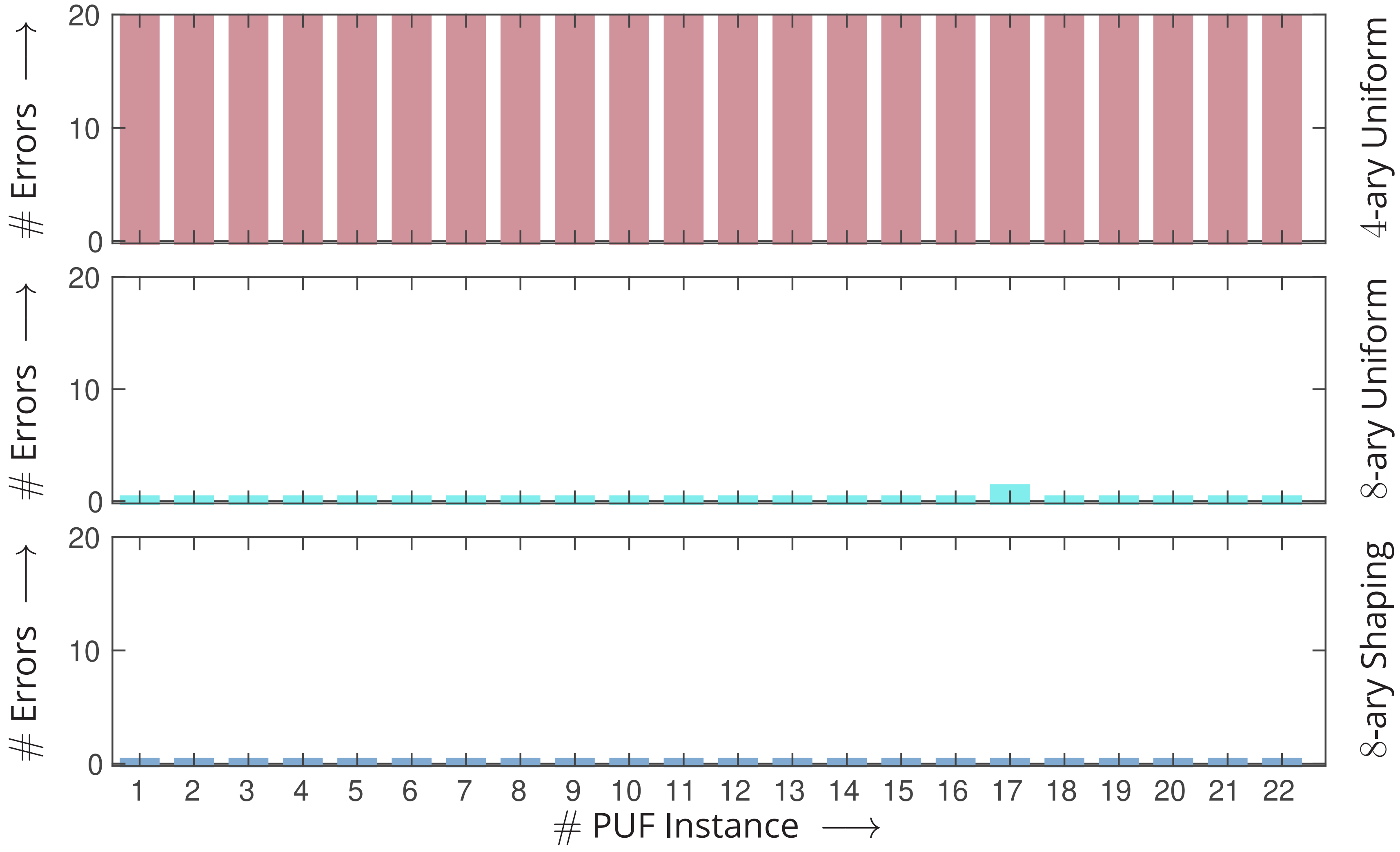
FPGA Implementation (II)

Number of Word Errors: per 70,000 readouts

- PUF nodes: 1024

  mess. length: 1792

  rate: $R = 1.75 \ \left[\frac{\text{bit}}{\text{node}}\right]$
- $S = 1$



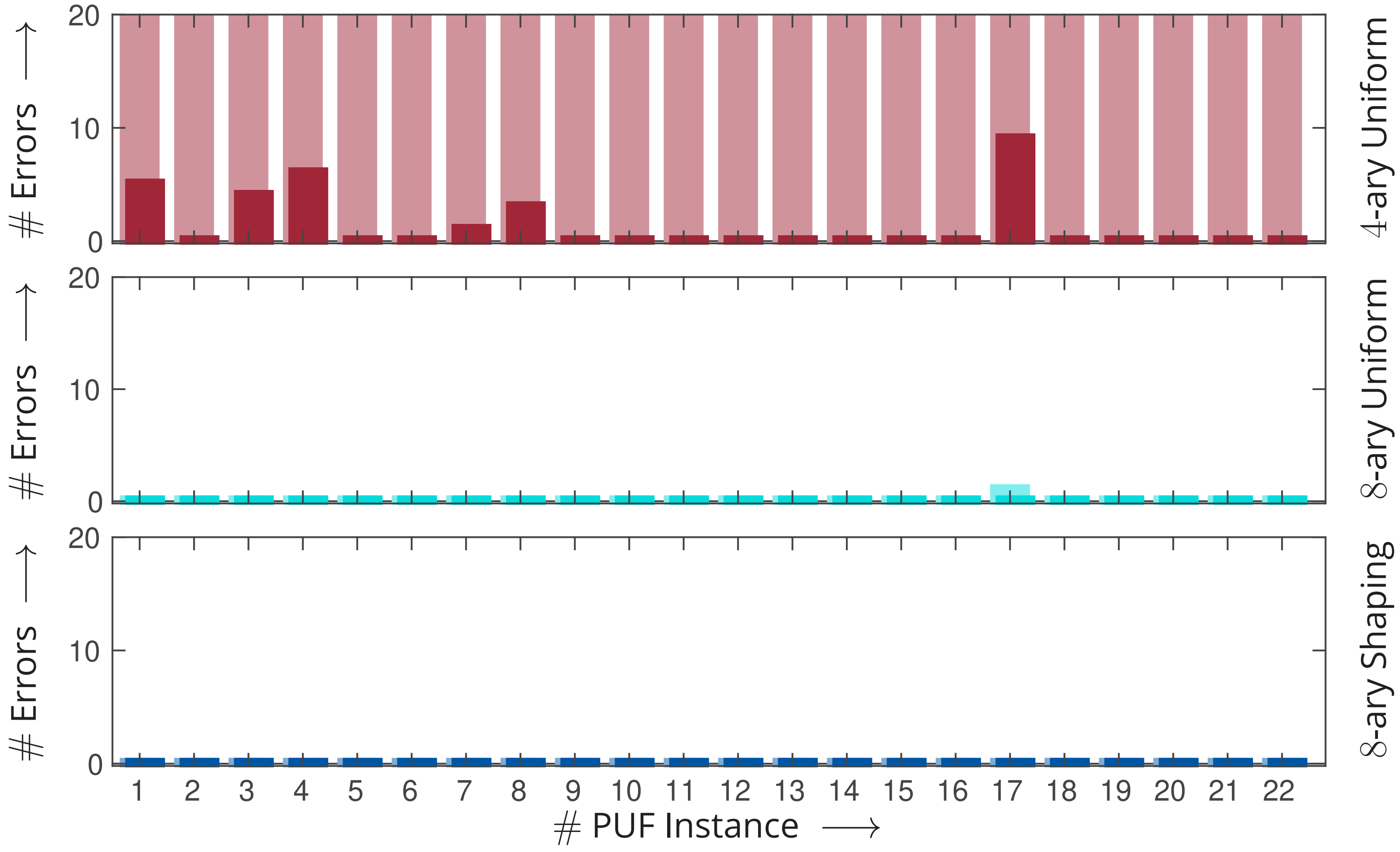
FPGA Implementation (II)

Number of Word Errors: per 70,000 readouts

- PUF nodes: 1024

  mess. length: 1792

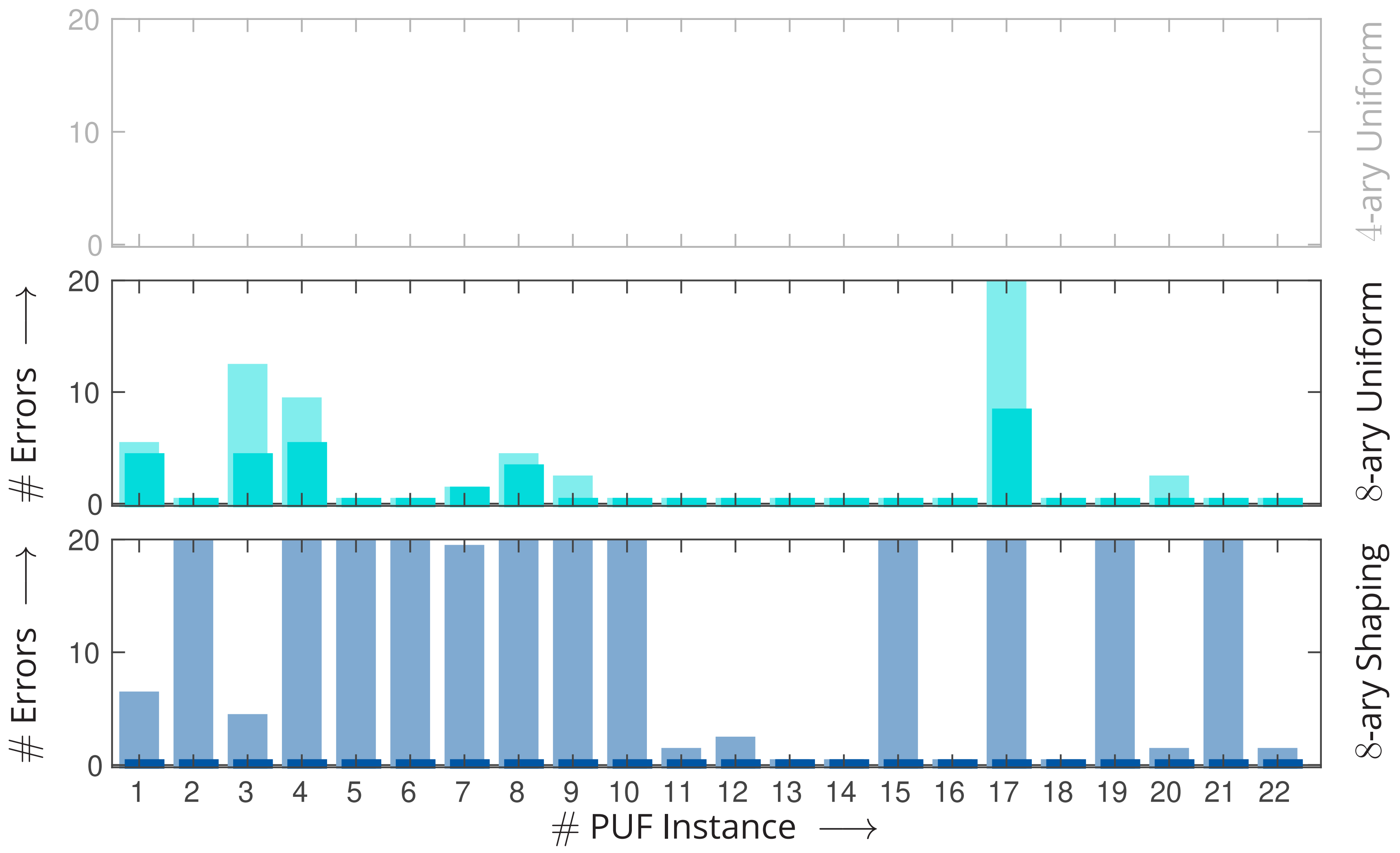
  rate: $R = 1.75$ $\left[\frac{\text{bit}}{\text{node}}\right]$
- $S = 1$ and 4



FPGA Implementation (II)

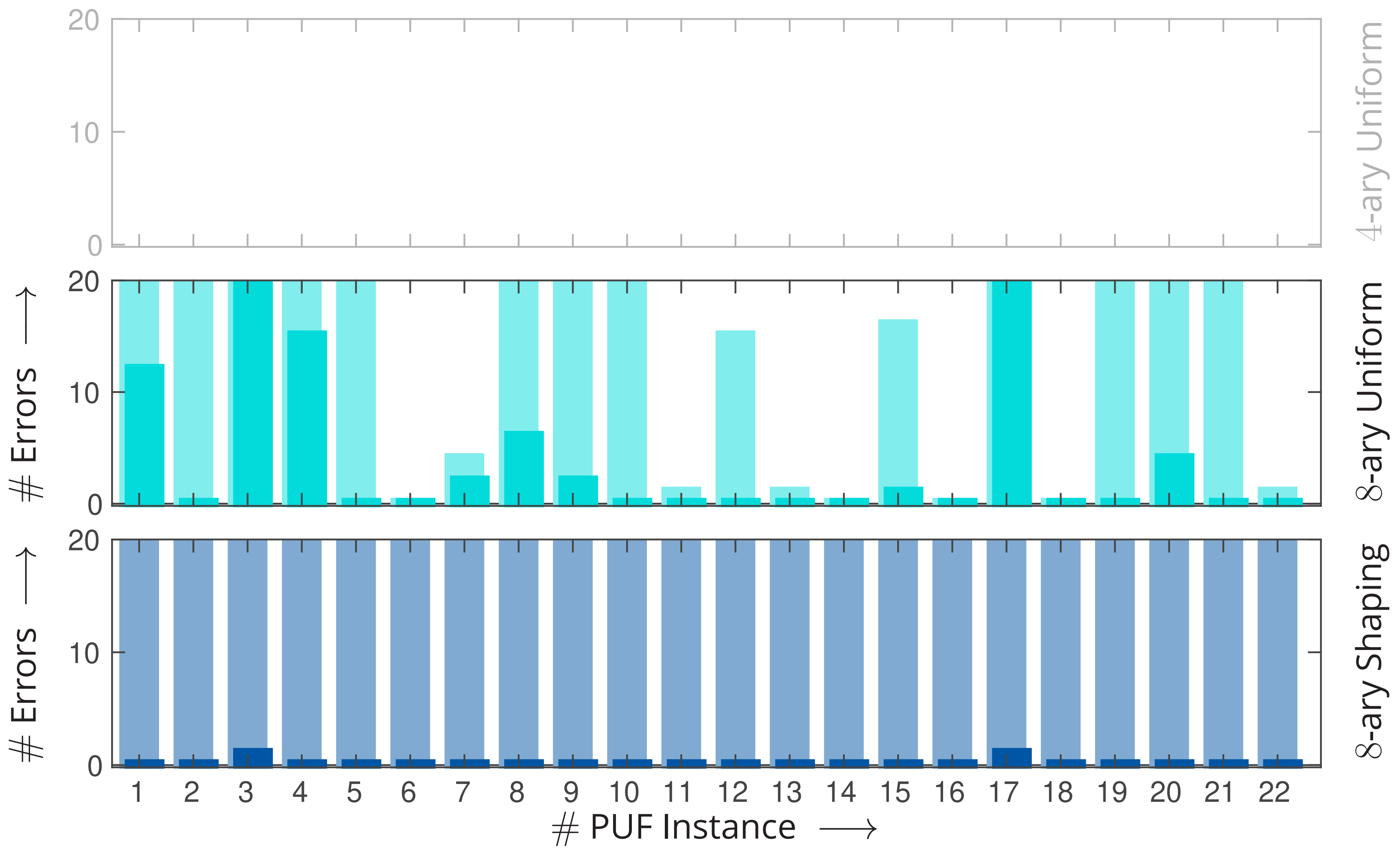
Number of Word Errors: per 70,000 readouts

- PUF nodes: 1024
  mess. length: 2048
  rate: $R = 2.00 \; [\frac{\text{bit}}{\text{node}}]$
- $S = 1$ and 4



Number of Word Errors: per 70,000 readouts

- PUF nodes: 1024
  mess. length: 2304
  rate: $R = 2.25 \; [\frac{\text{bit}}{\text{node}}]$
- $S = 1$ and 4



Summary and Outlook

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Error Correction for PUFs:

- utilizing the analog readout is rewarding
- PUF model: digital transmission with randomness at the transmitter
- design of coded modulation and shaping techniques ⇒ *increase in rate per PUF node*
- design of suited helper data ⇒ *increase in reliability*

Further Directions:

- here: Gaussian model for signal and error ⇒ *adaptation to real-world data*
- here: (silicon) PUF as hardware device ⇒ *application to “channel PUFs”*
- here: practical designs (coded modulation / helper data) ⇒ *fundamental finite-length limits*

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